A Multiattribute Auction Procedure and Its Implementation

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Abstract

A multiattribute auction procedure that allows bidders to make informed bids without knowledge of the bid-taker's preferences is presented. The procedure can be extended to allow the bid-taker to differentiate among the bidders, place them into categories, and formulate different requirements for each category. The procedure is adapted to reverse auctions and it is illustrated with a simple procurement case. The implementation of the procedure in the IMARAS system and the initial results from the system use in experimental settings are discussed.

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1.   Introduction

Most auctions are single attribute, i.e. price. A bid-taker sets up an auction in which bidders engage in a bidding process to buy a product or service from the buyer. We consider here single-attribute reverse auctions (SARAS) in which the bidders want to sell a product or service to the buyer.

The use of single-attribute auctions has two types of requirements to be met:

1. Auctioned goods have to be completely specified, with the exception of one attribute (price). The purpose of the bidding process is the specification of this attribute's value; and
2. For the bid-taker, the bidders do not differ in terms of their characteristics, i.e., there are no bidders who are more desirable than others.

The first requirement allows for the complete description of the auction with the use of one variable. This variable and its preferential direction are known to all participants; the direction is the same for the bidders and opposite for the bid-taker. In effect both the bid-taker and the bidders know what is better for them and that their interests are strictly opposite.

The second requirement complements the first in that bidder information does not need to be included in the auction description. The interest of the bid-taker is to obtain the best price for a given item irrespective of who the seller is. The result is that bidder may bid anonymously and does not need any information except for the bids.

In many real-life situations either one or both requirements are not met. A survey by Ferrin and Plank [1] found that over 90% of purchasing managers based their decisions on both price and non-price variables (e.g., durability, service, lead-time, and trust). Reverse auctions are used in purchasing, which consumes about 70% of corporate revenue. Reverse auctions have been shown to achieve an average gross savings of 15-20 per cent [2], contributing hundreds of millions of dollars in savings [3, 4].

The consideration of attributes other than price, describing the item and/or the bidders violates at least one of the principles. This leads to the modification of the auction selection, the auction itself, and/or winner determination.

The modifications include: (1) pre-selection of bidders so that only bidders who are known to meet the additional criteria are included; (2) giving incumbents an advantage because their qualifications are known; and (3) the use of disclaimers such as "the lowest bid may not be awarded the contract" [5-7]. The results of such auction modifications are mixed because of collusion and selection of inferior offers [8, 9]. In some situations the process becomes an auction in name only, as is the case with an auction in which neither the winner nor any other bidder is awarded the contract.

Rather than aggregating multiple attributes into one so that a single attribute auction can be used, it is possible to design a multiattribute auction. There have been several multiattribute reverse auctions (MARAS) models formulated, including models which give bidders all information that the buyer uses in order to analyze and compare bids [10]. This approach is unacceptable when buyers do not want to disclose their preferences for strategic, competitive, or other reasons [11, 12].

Another option rests on the assumption that all attributes can be expressed in monetary terms so that only two items need to be considered: (1) price, and (2) monetized attributes, which typically represent costs—for the sellers and value (income)—for the buyer. When an assumption is added that these two terms are monotonic and the buyer compares bids using the difference between value and price, then the sellers can determine the buyer's preferential order of the alternatives.

The attribute monetization methods have been widely implemented and tested [e.g., 5, 13, 14], and they are considered a standard in the auction literature [12]. The limitation of this method is the underlying assumption that: (1) the preferential direction of the monetized attributes is the same for all bidders and opposite for the bid-taker, and (2) there is a single function which transforms all attributes into money. The assumption deals with the attributes of the item but not the bidders, hence it cannot be used when the bid-taker wants to treat the bidders differently. It is also problematic if preferentially non-

monotonic attributes, such as brand, shape and color are considered.

The purpose of this paper is to present a MARA procedure which allows bidders to make informed bids with no knowledge of the bid-taker's preferences. The procedure can be extended to allow the bid-taker to differentiate among the bidders, classify them into categories, and formulate different requirements for each category. Our focus is on designing a feasible auction mechanism which allows for the achievement of efficient winning bids. The procedure allows for trading off the efficiency of the outcome with the efficiency of the process.

Note that although the procedure is described and illustrated for MARAs, it can easily be modified for standard multiattribute auctions.

The paper has four more sections. In Section 2 the multiattribute auction process is introduced through an illustrative example. In Sections 3 and 4, the proposed procedure and its extensions are described. First, the concept of bid limits based on the reservation levels is introduced, then the limit-sets are described and their use in MARA explained. The MARA system (IMARAS), in which the procedure is implemented, is given in Section 5.

2. Multiattribute auctions

Single-attribute auctions rely on the fact that the bidders know what is better for them and for the bid-taker, and that their interests are strictly opposite. In the case of SARA this means that every bidder knows that any other bidder prefers higher price and the buyer prefers lower price and the buyer knows that every seller prefers higher price.

2.1 Attribute aggregation

We consider the situation in which the buyer does not provide the bidders with information that would allow them to compare different alternatives and evaluate bids made by other bidders. This complicates the process because bidders do not know if a new bid is better for the buyer than an earlier bid.

Let's consider a very simple case in which the revenues (or profits) are functions of two attributes: price (p) measured in millions of dollars and lead time (l) measured in weeks. Let's assume that:

- Buyer's (B) profit depends on these two attributes and her revenue function is: $b = 30 - p - 0.4l^2$.
- Seller's 1 (S_1) revenue function that depends on these two attributes is: $s_1 = 1.2p + 4.2l - 4$.
- Seller's 2 (S_2) revenue function is: $s_2 = 2.3p + 2.3l - 5$.

Figure 1 illustrates selected revenue functions of the buyer and of the two sellers in the bid space.
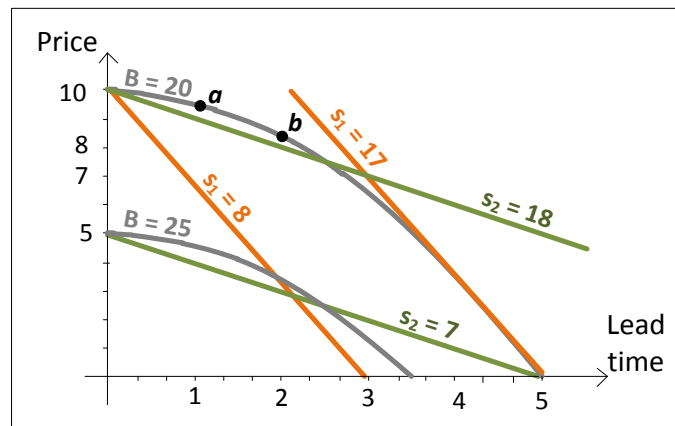

Fig. 1. Revenue functions of the buyer and two sellers.

Every combination of price and lead time which corresponds to a point lying on the same revenue

curve (line) yields the same revenue (e.g., points a and b yield the same profit of 20 for B). Alternatives yielding the same revenue for one participant (buyer and/or seller) typically yield different revenue for other participants.

2.2 Bidders' dilemma

The difficulties of making bids in multiattribute auctions are illustrated in Figure 2. Let's assume that seller S_1 made bid $b_1=(p=8; l=5)$ with revenue $s_1 = 24.9$. For the buyer B this offer yields revenue $b =12$ and for S_2 the revenue is $s_2 = 24.9$.

Bid b_1 may be shown to seller S_2 but he may have difficulties in making a bid that is better for B than b_1. For S_2 to be sure that his bid is better than the previous bid, he has to decrease the value of both attributes. (Note that this requires an assumption that the buyer's preferences on the attributes are opposite, albeit not necessarily strictly opposite, to the seller's preferences.)
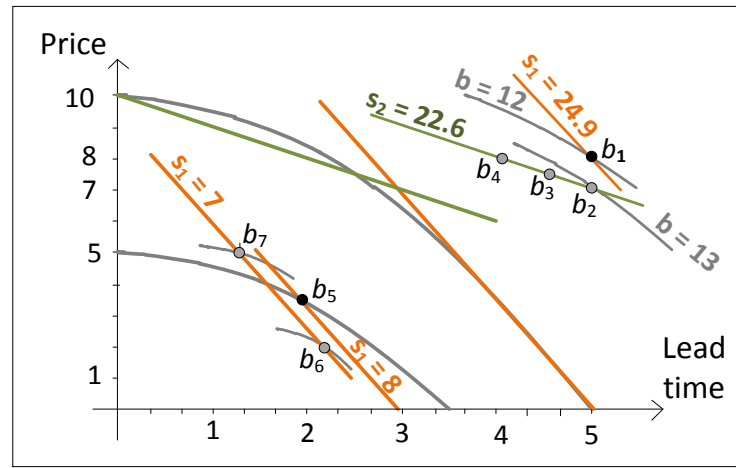

Fig. 2. The bidder's dilemma.

Decreasing both attribute values may be a move that is certainly better for the buyer than b_1, but it may be a strategically ineffective one. This is because such moves do not allow for making trade-offs, hence they may result in a winning bid that is inefficient, i.e., there is a bidder who would make a bid that is better for the buyer but could not because the bid was not admissible.

After S_1 made bid $b_1 = (8; 5)$, S_2 may consider the following three potential bids: $b_2 = (7; 5)$; $b_3 = (7.5; 4.5)$ and $b_4 = (8; 4)$. These three bids differ from b_1 in that either one is subtracted from only one attribute, or 0.5 is subtracted from two attributes.

Seller S_2 is indifferent between b_2, b_3, and b_4 but the buyer *and* S_1 are not. If S_1 knew about the buyer's revenue function, then he should bid b_2. This bid may allow S2 to bid later with b_3 or b_4, depending on what S_1 bids.

The problem arises when the sellers do not know what the buyer's revenue function is. Every seller has to rely solely on the knowledge of his own revenue function and the bids made by other sellers. In such a situation the bidders may not only be unable to optimize their strategies, but even make bids that are worse than their earlier bids. Such bids are inadmissible and they would be rejected by the buyer; the problem is that the sellers have no ability to distinguish between admissible and inadmissible bids.

Table 1. Bids and their corresponding revenue values.

Bider (bid)	Price	Lead time	Revenue B	Revenue S1	Revenue S2
S_1 (b_1)	8	5	12	26.6	24.9
S_2 (b_2)	7	5	13	25.4	22.6
S_2 (b_3)	7.5	4.5	14.4	23.9	22.6
S_2 (b_4)	8	4	15.6	22.4	22.6
S_1 (b_5)	3.7	1.8	25	8	7.7
S_1 (b_6)	5	1.2	24.4	7	9.3
S_1 (b_7)	2	2.2	26.1	7	4.7

3. Bid limits

We describe the proposed MARA procedure using the example introduced above. The issue of inadmissible bids like bid b_7 is addressed by the introduction of *limits*. Limits are bounds set up for the attribute values and they help sellers to make only progressive bids, that is, bids which are better for the buyer. In this way we give the bidders information which makes bidding in multiattribute auction similar to bidding in the single-attribute auction.

At each point in time (or round) there may be one or more sets of limits. One set defines limits on every attribute. In Figure 3, three rounds are indicated. In Round 1 the best bid is b_1; in Round 2 it is b_2; and in Round 3 the best bid is b_5.
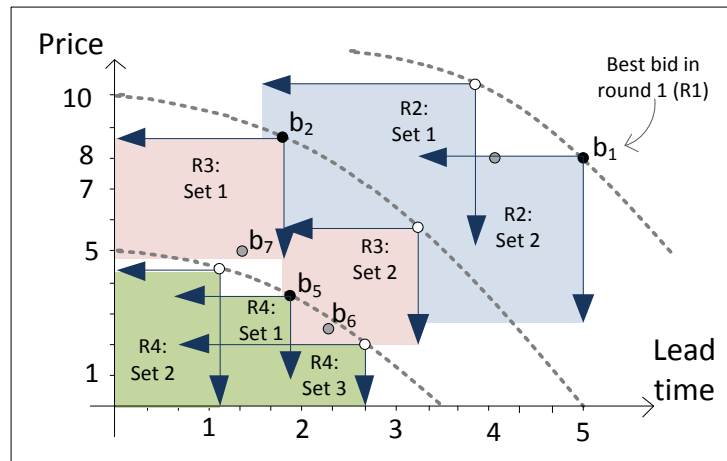

Fig. 3. Bidding in the limit-sets.

The limits for each round are determined based on the best bid in the previous round and additional parameters set up by the buyer (e.g., the number of limit sets calculated for each round). This means that there may be more than one set of limits in each round.

For Rounds 2 and 3, two sets per round are shown in Figure 3; they are indicated by rectangles bounded by arrows; every bid in the rectangle is admissible. Three limit sets are determined for bidding in Round 4.

Bids outside the limit-sets are inadmissible. Observe that the limits assure that, after S_2 made the winning bid b_5 in Round 3, S_1 cannot make b$_7$ because this bid is outside of the limits. Bid b_6, however, can be made. The buyer's revenue function is now shown (Fig. 3) with dotted lines to indicate that the bidders do not know it; for them it is sufficient to know the limits given in Table 2.

Table 2. Limit-sets

Round no.	Set no.	Price	Lead time
Round 2	Set 1	$p \leq 8$	$l \leq 5$
	Set 2	$p \leq 10.3$	$l \leq 3.8$
Round 3	Set 1	$p \leq 7$	$l \leq 5$
	Set 2	$p \leq 5.8$	$l \leq 3.3$
Round 4	Set 1	$p \leq 3.7$	$l \leq 1.8$
	Set 2	$p \leq 4.3$	$l \leq 1.1$
	Set 3	$p \leq 2$	$l \leq 3.7$

In each round, every bidder may choose the set of limits that best suits his interests. Every bidder, however, has to observe at least one set of limits. This means that at the beginning of Round 2, the sellers obtain two limit sets (see Table 2) which they have to follow. Their bids have to be such that: (1) either the price is not greater than 8 and the lead time not greater than 5; or (2) the price must be not greater than 10.3 and the lead time not greater than 3.8.

4. Procedure

The proposed MARA procedure builds on the one proposed by Bellosta et al. [15, 16], [17], and Teich et al. [18]. Bellosta, et al. (2008) and Brigui-Chtioui et al. (2010) auction design for multiattribute reverse auctions relies on the notion of reservation levels for which constructing the preference aggregation method is used. In this paper we also use the buyer's reservation levels for auction design. The key difference is the space in which these levels are constructed. While in both procedures the levels originate in the utility space, we transform the reservation from the utility space to the space of alternatives. This has an important and desirable impact on the information feedback: similarly to Teich et al. [18], any information conveyed to the sellers refers to the space of alternatives.

4.1 Problem representation

There are l sellers (S_l, $l=1, ..., L$) interested in selling an item $\boldsymbol{x} = [x_n, n=1, .., N]$ characterized by N attributes which need to be determined in an auction.

We assume that the buyer is interested in all N attributes and that she has constructed a utility function $u_B(\boldsymbol{x})$, (or any measure defined on the attributes and preferences). Without loss of generality, we assume that: (1) the buyer prefers smaller attribute values over greater ones (e.g., lower price and shorter lead time); and (2) the bidding space is represented by a closed set X. This means that every attribute has its upper limit (e.g., market price) and lower limit (e.g., zero or some minimum possible value). The upper limits thus represent the maximum values $[x_l^M]$ acceptable for the buyer and they correspond to the minimum acceptable utility $u_B(\boldsymbol{x}^M)$.

In what follows, we also assume that set X is discrete, i.e., the number of feasible bids is finite. The procedure can be applied to continuous variables but its introduction is simpler for discrete ones which are typical for real-life situations.

In the example (Fig. 3) we constructed two sets for R2 and R3 and three sets for R4. The number of

sets that is constructed depends on three aspects:

1. The shape of the utility function (i.e., linear, convex, non-convex). In general, more complex functions require the construction of more limit-sets;
2. The number of admissible alternative bids that are removed from the admissible bidding space. The fewer alternative bids that the buyer agrees to be removed, the more sets are needed; and
3. The acceptable degree of auction process difficulty; more limit-sets increase the difficulty.

The number of sets may be fixed for the auction or it can be controlled and depend on, e.g., time and distance from the best possible alternative. For simplicity, we assume here that the number (K) is fixed. Hence, in every round t ($t = 1, ..., T$), X_{kt} ($k = 1, ... K$) sets are constructed.

The auction begins with the bidders being given the highest reservation values of the attributes, i.e., the admissible bids are x: $x \in X$. After the initial bids are submitted the subsequent activities are the same for every round.

4.2 Limit-set construction

Let x^*_t be the bid which yields the highest value of the buyer's utility, i.e., $u^*_t = u(x^*_t)$. Point x^*_t is the first one to be used to determine the limit-set for the next round, i.e., $X_{t+1} = \{x_{t+1,j} \geq x^*_{tj}\}$. The remaining k-1 points are selected in the way which assures that the distribution of the points covers the widest possible range of the feasible alternatives in this round [19].

The limit-set construction process for $K = 2$ is illustrated in Figure 4. Three bids were made at round t (shown as stars). The best bid x^*_t yields utility u^*_t. The search for a distinct alternative which yields the same utility results in x_a.

The two alternatives x^*_t and x_a can be used to generate limit-sets. The effect of selecting alternatives yielding the same utility may allow the bidders' to approximate the bid-taker's utility.

It is also possible that the number of such alternatives is smaller than the number of limit-set required for the particular auction. Therefore, the procedure searches for alternatives that are within a certain distance from the alternatives yielding u^*_t. In Figure 4 this search results in the selection of x_b. Alternatives x^*_t and x_b are used to construct $X_{t+1,1} = \{x_1 \geq x^*_{1,t}, x_2 \geq x^*_{2,t}\}$ and $X_{t+1,2} = \{x_1 \geq x_{1,b}, x_2 \geq x_{2,b}\}$, respectively.
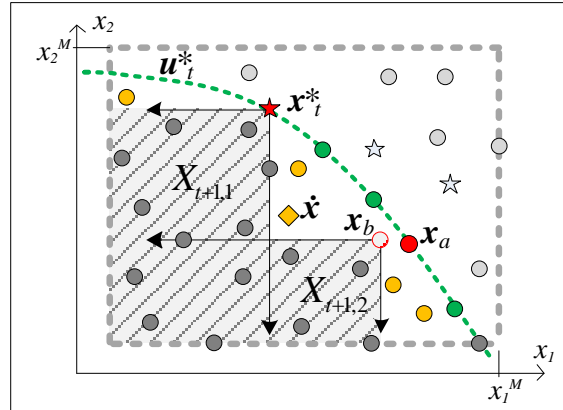

Fig. 4. Construction of the limit-sets.

Sets $X_{t+1,1}$ and $X_{t+1,2}$ are presented to the bidders who are asked to make admissible bids. If bids are submitted, then the best bid is selected, its utility calculated and the process continues as shown in Figure 4.

The distance (d) used for the modification of the alternatives used to construct the limit-sets is one of the auction design parameters. The greater the distance, fewer rounds are required but then there is a

greater possibility of the winning bid being inefficient. The use of k limit-sets typically will result in some alternatives being inadmissible, despite the fact that they yield utility greater than u^*_t. Among these alternatives (shown in yellow in Fig. 4) there is $\dot{x}$ (diamond). If in round $t+1$ no bids are made, then x^*_t becomes the winning bid. It is possible, however, that a bidder would offer $\dot{x}$, if $\dot{x}$ was admissible. If that were the case, then the procedure causes that the auction ends with an inefficient alternative.

4.3 Bidder differentiations

We have mentioned that one reason for using multiattribute auctions is the differences among the bidders which are important for the bid-taker. This means that bidders have different characteristics and some of them are preferred over the others.

The proposed procedure allows for the distinction among bidders when the following two conditions are met:

1. The bid-taker can partition all suppliers into exclusive groups comprising suppliers with the same or very similar characteristics so there are no differences among suppliers from the same group; and
2. The bid-taker can measure the differences between each supplier group in terms of the utility; the bid-taker is indifferent between members coming from different groups if the bid utility from a less desirable member is greater than the bid from a more desirable member.

Let H denote the number of groups; $h = 1, ..., H$ and let's assume that the lower the group index the lower the utility assigned to this group. That is, the members from group h are less desirable than the members from group $h+1$.

Let $w_1, ..., w_H$ ($w_h > w_{h+1}$) be utility values required from making tradeoffs between bidders. Condition (2) states that the desirability can be measured with utility, so that bidders S_h (from group h) and S_{h+1} (from group $h+1$) are equally desirable if the utility of the bid made by B_h is equal to the utility of S_{h+1} bid adjusted by w_h.

In the procedure with H different types of bidders, the process of the limit-set construction is divided into H sub-processes. In each sub-process the steps are essentially the same; the only difference is in the computation of the utility value of the best bid in round t ($t = 2, ..., T$). For a uniform group of bidders (discussed in Section 3.2) the utility value that is used as the basis for the limit-set construction is the actual bid utility, i.e.,

$$u^*_t = u(x^*_t);$$

If there are H types of bidders, we need to construct $H \times K$ of limit-sets. Instead of the above formula we use the following formula:

$$u^*_{th^*} = u(\dot{x}_{th^*}) = \max_{h=1,...,H} \{ u(x_{th}) / w_h \},$$

where $\dot{x}_{th^*}$ is the bid yielding the highest utility value after the adjustment made that account for the group differences.

If the coefficients w_h ($h=1, ..., H$) are normalized so that $w_H = 1$, then the less desirable sellers have to make better bids in order to be considered as good as the bids made by the more desirable sellers (i.e., $u_h < 1$, $h < H$).
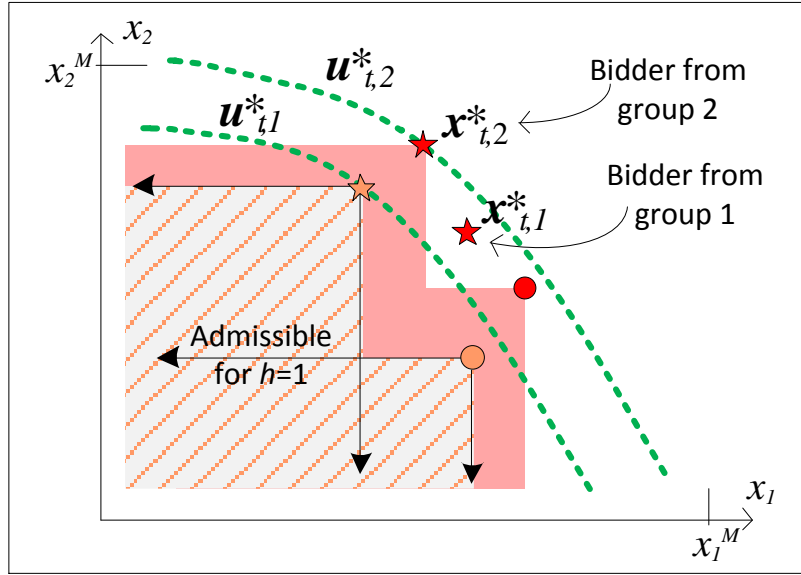
Fig. 5. Limit-sets for two types of bidders (*H*=2).

An example of two types of bidders (*H*=2) is shown in Figure 5. A seller of the best type (*h*=2) made a bid $x^*_{t,2}$ which utility is $u^*_{t,2}= u(x^*_{t,2})$. The best bid by a seller from group 1 is $x^*_{t,1}$. The utility of this bid $u(x^*_{t,1})$ is higher than $u(x^*_{t,2})$, but it is lower than $u(x^*_{t,2})/w_1$. Therefore, $\dot{x}_t = x^*_{t,2}$ and $u^*_{t,2}$ is used as the basis for the construction of the limit sets for the next round.

The limit-set for members of group 2 are the same as in the example shown in Figure 4. For group 1 the required utility value is calculated so that it is equivalent to the utility of group 1, i.e., $u^*_{t1} = u^*_{t2} / w_1$. Selecting two points yielding this utility value and distorting one of them allows for the construction of limit-sets containing admissible bids for members of group 1. The admissible area is shown in Figure 5. The shaded area contains the bids which are admissible to group 2 members but inadmissible to group 1 members. The latter need to make bids which yield a higher utility.

The bidder-differentiation process needs not rely on the utility; instead selected attributes can be used. This allows viewing some bidders as not reliable on time delivery and others on quality. If the buyer wants to take bids made by such sellers into account, then she needs to determine attributes on which these sellers must exceed other, more reliable sellers.

5. Implementation

The proposed procedure has been implemented on the Invite platform, which is a software platform for the development of multiattribute negotiation and auction systems. The IMARAS system is being been used to test the users' behavior under different conditions controlled by the above mentioned design parameters.

5.1 System features

IMARAS adopts the model-view-controller design pattern, which allows separation of the different auction procedures and the user interface. It supports several types of auction settings, including:

- Disclosure of bids to bidders: only bidder's own bid is displayed, both own and winning bids are displayed, or all bids are displayed;
- Bidding process: continuous (asynchronous bidding) or round-based (synchronous bidding); with rounds being defined by time (e.g., number of minutes or hours) or defined by a rule (e.g., number of

submitted bids).

The design pattern used in IMARAS allows for decoupling of the interface from the engine. This helps us in conducting experiments with different mechanisms that have a very similar user interface.

IMARAS main screen is shown in Figure 6; it is the bidding screen of a round-based auction in which the bidder can see her own bids and the winning bids.
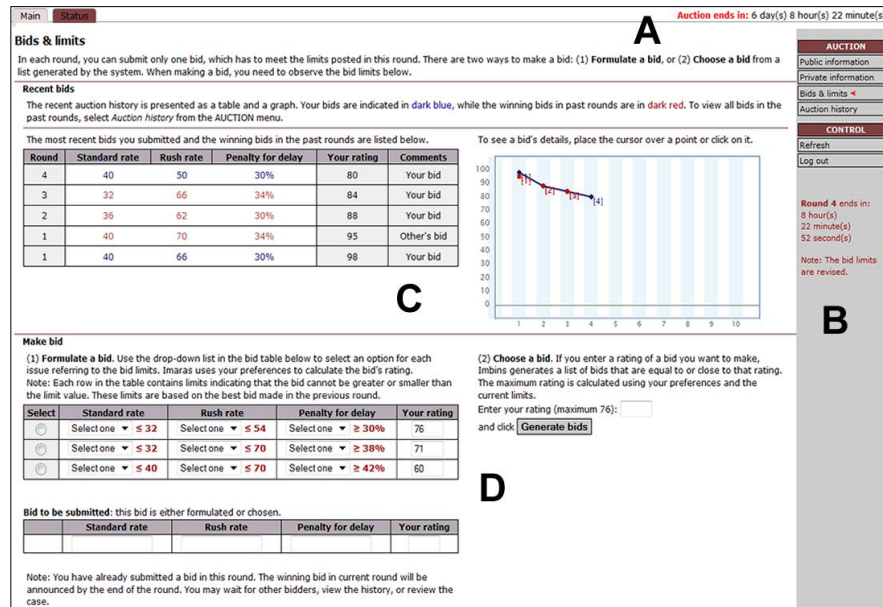

Fig. 6. IMARAS bidding screen.

The interface above has four main components. The auction clock (A) shows the time from the beginning of the auction and the time left to the deadline. The round number and clock (B), the latter is reset at the beginning of every round. In section C, the user's past bids and winning bids is shown in both tabular ad graphical forms. Section D contains information about the limit sets (table on the left-hand side) and the tools to generate alternative bids and select one of them. The user can provide values for each attribute; following it the system calculates the user's utility for the bid. Alternatively, the user may specify the desired utility value and request that the system generates alternatives yielding this value or one close to it. From the generated table with alternatives the user selects one and submits it as her bid.

5.2   Procedure illustration

The key elements of the procedure are illustrated with a transportation service procurement scenario. A milk producer (the buyer) sets up an auction to outsource the service described by three attributes: *standard rate* (SR), *rush rate* (RR) for rush orders and *penalty for delay* (PD) for late deliveries. The weight of each attribute is private while the attribute value range and reservation levels are public to all bidders (see Table 3).

The auction is round-based with the disclosure of bidder's own bids and the winning bids in previous rounds. The system calculates limit-sets at the beginning of each round based on the winning bid made in the previous round and the parameters set up by the buyer. In this case, the parameters are:

- Reservation levels on each attribute (see Table 3);
- Increment: $4/kl for SR and RR, 4% for PD;
- Maximum number of limit-sets provided to the bidders: 3 (if they all yield the same utility for the buyer, one of them will be modified); and
- Round duration and auction length: 5 minutes/round and 50 minutes/auction.

Table 3. Attributes, weights and reservation levels

Attribute	SR	RR	PD
Weight	0.45	0.15	0.40
Value range	$20 ~ 40/kl	$50 ~ 70/kl	30 ~ 50%
Reservation levels	$40/kl	$70/kl	30%

Knowledge of the current limit-sets is sufficient to make bids. An allowable bid is one that conforms to at least one of the limit-sets. The auction will close if one of the following conditions is met:

- No more than one bidder bids in a round; or
- The closing time is reached.

Table 4 shows four rounds of the auction from one bidder's view (the best bid in each round is italicized).

Table 4. Auction process for one bidder

Round	SR	RR	PD	Revenue	Bid
R1	36	66	38%	78	Own
	32	*66*	*38%*	*68*	*Other*
R2	32	62	46%	57	Own
	24	*54*	*34%*	*44*	*Other*
R3	*32*	*58*	*50%*	*45*	*Own*
R4	24	54	38%	38	Own
	20	*62*	*42%*	*24*	*Other*

The process demonstrates how the bidder was making trade-offers among the three attributes. For example, in R3 she bid with a lower RR than her own bid but higher than the winning bid in R2, raised PD and maintained SR (higher than winning bid). As the buyer weighted SR and PD much higher than RR, the bidder won in R3.

An example of the auction process with four bidders and eight rounds is shown in Figure 7. This type of chart is shown to buyers. The sellers' view may be different; this depends on the design parameters, including representation of only own bids, own and winning bids, or all bids.
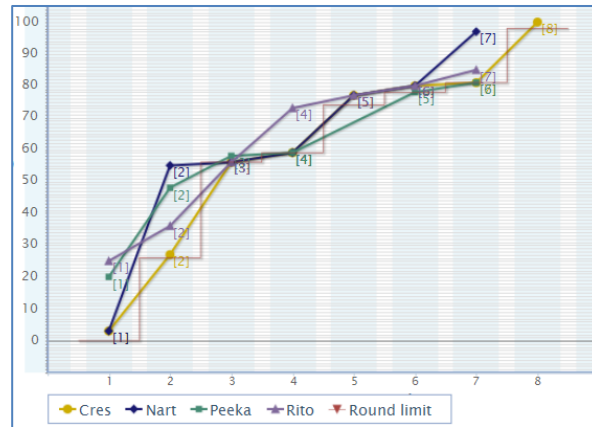
Fig. 7. Buyer's view of the auction process.

The bids are depicted on the chart with the buyer's revenue (from 0 to 100) and the round number. The step line indicates the utility of lower-bound limits in each round, and thus the bids should not be lower than it and the highest one is the winning bid in that round.

Acknowledgements

References

[1]   B. G. Ferrin and R. E. Plank, "Total Cost of Ownership Models: An Exploratory Study," *Journal of Supply Chain Management,* vol. 38, pp. 18-29, 2002.

[2]   L. Cohn, "B2B: The Hottest Net Bet Yet?," *Business Week,* 2000.

[3]   B. Peleg. (2003, The Value of Procurement via Online Bidding. *Whitepaper,* 3. Available: http://www.crm2day.com/crm_articles/articles-93.php

[4]   S. M. Wagner and A. P. Schwab, "Setting the Stage for Successful Electronic Reverse Auctions," *Journal of Purchasing and Supply Management,* vol. 10, pp. 11-26, 2004.

[5]   M. Bichler and J. Kalagnanam, "Configurable Offers and Winner Determination in Multi-attribute Auctions," *European Journal of Operational Research,* vol. 160, pp. 380-394, 2005.

[6]   R. Engelbrecht-Wiggans, E. Haruvy, and E. Katok, "A Comparison of Buyer-determined and Price-based Multi-attribute Mechanisms," *Marketing Science,* vol. 26, pp. 629–641, 2007.

[7]   T. Schoenherr and V. A. Mabert, "Online Reverse Auctions: Common Myths versus Evolving Reality," *Business horizons,* vol. 50, pp. 373-384, 2007.

[8]   W. Elmaghraby, "Auctions and Pricing in E-marketplaces," in *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-business Era*, D. Simchi-Levi, D. Wu, and M. Shen, Eds., ed Norwell: Kulwer, 2004.

[9]   E. Katok and A. Wambach, "Collusion in Dynamic Buyer-Determined Reverse Auctions," *Management Science,* forthcoming, 2011.

[10] C. H. Chen-Ritzo, T. P. Harrison, A. M. Kwasnica, and D. J. Thomas, "Better, Faster, Cheaper: An Experimental Analysis of a Multiattribute Reverse Auction Mechanism with Restricted Information Feedback," *Management Science,* vol. 51, pp. 1753-1762, 2005.

[11] B. Burmeister, T. Ihde, T. Kittsteiner, B. Moldovanu, and J. Nikutta, "A Practical Approach to Multi-attribute Auctions," in *13th Workshop on Database and Expert Systems Applications (DEXA)*, 2002, pp. 670-674.

[12] D. C. Parkes and J. Kalagnanam, "Models for Iterative Multiattribute Procurement Auctions," *Management Science,* vol. 51, pp. 435–451, 2005.

[13] Y. K. Che, "Design Competition through Multidimensional Auctions," *The RAND Journal of Economics,* vol. 24, pp. 668-680, 1993.

[14] S. Strecker and S. Seifert, "Electronic Sourcing with Multi-attribute Auctions," in *HICSS 37*, Hawaii, 2004.

[15] M. Bellosta, S. Kornman, and D. Vanderpooten, "A Unified Framework for Multiple Criteria Auction Mechanisms," *Web Intelligence and Agent Systems,* vol. 6, pp. 401-419, 2008.

[16] M. J. Bellosta, I. Brigui, S. Kornmanand, and D. Vanderpooten, "A Multicriteria Model for Electronic Auions," in *ACM Symposium on Applied Computing (SAC'04)*, 2004, pp. 759–765.

[17] I. Brigui-Chtioui and S. Pinson, "A Variable Bid Increment Algorithm for Reverse English Auction," in *Progress in Artitficial Economics*, 2010, pp. 41-51.

[18] J. E. Teich, H. Wallenius, and J. Wallenius, "Multiple Issue Auction and Market Algorithms for the World Wide Web," *Decision Support Systems,* vol. 26, pp. 49–66, 1999.

[19] G. E. Kersten, S. Wu, and T. Szapiro, "A Procedure for Multiattribute Reverse Auctions with two Strategic Parameters " Concordia University, Montreal, 2010.