InterNeg

Multi-attribute Procurement Auctions: Efficiency and Social Welfare in Theory and Practice

Gregory E. Kersten

InterNeg Research Centre, Concordia University, Montreal

Abstract

Quasi-linearity of preferences is one of the standard assumptions in auction theory. This assumption is of particular significance in multi-attribute reverse auctions which are used in procurement. This paper presents an analysis of this assumption and its implications. Building on observations of scholars in economics and decision sciences who note that in practice such preferences may be rare, it shows that in procurement of services and goods that are to be produced, price is often interrelated with costs. When preferences can be represented with convex or concave utilities, the alternatives in which the buyer's surplus is maximized are different from those that maximize social welfare. The result is that reverse auctions may cause a significant loss of social welfare, which may be of particular significance for public organizations. The analysis of concave efficient frontiers in the utility space which are the result of concave and linear utility functions, shows that it is possible to determine alternatives for which social welfare is greater than for the alternative which is the winning bids. If the winning seller is willing to share the increase in utility with the buyer who faces a loss, then these alternatives can produce for both the buyer and the seller, utility values that are higher than produced by the winning bid.

Keywords: Bidding, procurement, reverse auctions, multi-attribute actions, utility, efficiency, social welfare, multi-bilateral negotiations

http://interneg.org/

1. Introduction

My father said: "You must never try to make all the money that's in a deal. Let the other fellow make some money too..." (John Paul Getty about his father, Jean Paul Getty, American billionaire oilman, 1892-1976)

Jean Paul Getty gave this advice because he considered reputation as an important deal-making attribute. If reputation is not important, then one may presume that taking as much of the value as possible is a rational and economically justifiable action. Auction theory is based on this presumption. The purpose of this paper is to show that this presumption should be questioned in reverse auctions, which are pervasive exchange mechanisms in the procurement of commodities, products and services. The paper also argues that reverse auctions may have negative rather than positive impact on markets and their participants and that giving some of the gained value back to the bidders may be beneficial.

Some auctioned goods and are uniform but many are heterogeneous; this is especially true in the case of procured services. These goods are described by price as well as non-price attributes (Ferrin and Plank, 2002; Plank and Ferrin, 2002). The number of attributes considered may range from 2 to 30 or more (Gupta et al., 2013; Iimi, 2013). Thus, there is thus a need for multi-attribute exchange mechanisms in which utility or other measures are used (see e.g., Che, 1993; Branco, 1997; Beil and Wein, 2003; Chen-Ritzo et al., 2005; Simon and Melese, 2011).

The two standard assumptions in auction theory are that: (1) market participants are risk-neutral; and (2) their utilities are represented with quasi-linear functions (Kalagnanam and Parkes, 2004; Parkes and Kalagnanam, 2005). In multi-attribute problems the second assumption is of particular importance; it means that the utility is the sum of two components: (1) price (called a numeraire), which is continuous, linear and independent of any other attributes, and (2) the valuation component, which is a convex function expressed in monetary terms. These assumptions assure that the auction is *allocative efficient* (i.e., maximizes social welfare) and its outcome is an *efficient solution* (i.e., the solution is Pareto-optimal). If the suppliers are rational agents and bid their true valuations, then these assumptions also assure that the winning bid *maximizes the buyer's utility* (i.e., surplus).

Quasi-linearity is a strong assumption because "it is not in general possible to model a well-behaved exchange economy as a transferable game" (Bergstrom and Varian, 1985, p. 222). Luce and Raiffa (1957, p. 168) observe that situations in which quasi-linear utilities "can realistically happen remains obscure". Another difficulty is specific to situations in which goods are not yet available; they are created only after they are contracted. In such situations it is natural for both the buyer and the suppliers/producers to view prices as being dependent on other attributes and vice versa. The attribute interdependence is especially likely in procurement of services. Ausubel and Milgrom (2006, p. 24) observe that the quasi-linearity assumption is very restrictive and add: "it requires that there is no effective budget limit to constrain the bidders and that the buyer, in procurement auction, does not have any overall limit on its costs of procurement. Although we have no data on how frequently these assumptions are satisfied, it appears that failures may be common in practice."

The limitations of quasi-linearity assumption may be the reason for a number of multi-attribute mechanisms that do not depend on this assumption (e.g., Sandholm, 2007; Bellosta et al., 2008; Karakaya and Köksalan, 2011; Adomavicius et al., 2012). Approaches that use decision analytic methods (e.g., MAUT, AHP, ELECTRE and SMART) for the construction of market participants' utilities do not distinguish between different attributes and they allow for interdependencies and trade-offs between all attributes, including price (Che, 1993; Bichler and Kalagnanam, 2005;

Kameshwaran et al., 2007; Gwebu, 2009). Additionally, hybrid approaches in which auctions are augmented with negotiations employ monotonic but not quasi-linear preferences (Pham et al., 2013).

If the quasi-linearity assumption is not met, then the construction of a strategy-proof efficient auction mechanism which produces a Pareto-efficient solution is not possible as the Gibbard-Satterthwaite impossibility theorem shows. This paper shows that auctions, which are based on preferences other than quasi-linear preferences may result in an efficient solution, but they are inefficient mechanisms (i.e., allocative inefficient). This may be of a lesser concern in a business organizations' procurement because if these organizations are primarily concerned with their own interests. Public organizations however, while responsible for individual transactions, are expected to contribute to creating societal welfare. If they use an allocative inefficient mechanism then they fail in their mission; they may get the best possible contract but in doing so they weaken the economy. The result is social welfare loss. Because multi-attribute reverse auctions have been widely adopted by public organizations their effects may be damaging. Revisiting the features and implications of (non)quasi-linear preferences is thus worthwhile.

Preferences of the market participants as well as budgetary and other constraints complicate multi-attribute auctions. Budget and other constraints are often necessary in procurement, however, their effect is that no efficient mechanism and strategy proof that meets these constraints can be constructed (Hurwicz and Walker, 1990). Constraints may result in the efficient frontier for quasi-linear preferences having similar features as the efficient frontier in the case of convex and quasi-convex preferences.

The importance of quasi-linear preferences in auction theory has been acknowledged; a number of theoretical models rely on them (see e.g., Beil and Wein, 2003; Chen-Ritzo, Harrison et al., 2005; David et al., 2006; Asker and Cantillon, 2008). It appears, however, that the assumptions underlying such preferences have not been discussed in sufficient detail for auction designers and users to understand the assumptions' requirements and the implications of their violation. Literature in which multi-attribute auctions have been proposed and which suggests decision analysis and the use of scoring function tends to assume quasi-linear preferences but ignores the difficulty of its attainment as well as the implications of its violation. This paper aims to address these limitations, show the potential losses of social welfare, and suggest alternative mechanisms.

Review of auction literature and an analysis of cases posted by companies offering online procurement platforms, discussed in Section 2, does not give insight into the losses of social welfare caused by inefficient auctions. The review as well as the assessment of the concrete multi-attribute procurement transactions, in which auctions were employed, indicates that such losses have not been considered. In order to explain the source of the losses an in-depth discussion of quasi-linear preferences is given in Section 3. Implications of such preferences are discussed in Section 4. A better understanding of the requirements for such preferences and their implications should allow practitioners to make informed decisions regarding the use of auction mechanisms.

Section 5 discusses social welfare losses due to attribute interdependency, differences in the preferential order, and the monotonicity of the scoring functions. Many decision-theoretic models rely on more general preferences, which in cases with two decision-makers lead to a convex feasible set in the utility space with a concave efficient frontier. This section shows that if the efficient frontier is concave, then it is possible to obtain an alternative that is preferred by buyers and by sellers over the winning bids.

The paper addresses the question of whether reverse auctions are suitable in transactions in which the assumption of quasi-linear preferences is violated. It shows in Sections 4 and 5 that

such violations may cause a significant loss in social welfare. This should be of concern to public organizations whose mission it is not only to obtain the best possible product or service at the lowest possible price but also to contribute to social welfare.

Section 6 concludes the paper. It outlines three types of exchange mechanisms which market participants may prefer over reverse auctions. It is not possible to maximize both social welfare and the buyer's surplus when preferences are not quasi-linear. However, it may be possible to determine alternatives, which improve both as opposed to winning bids, which are achieved in reverse auctions. Design of mechanisms for general preferences is beyond the scope of this paper, which, however, provides directions for such efforts.

2. Multi-attribute auctions and quasi-auctions

Many multi-attribute auction mechanisms have been designed but only some were implemented in software and empirically verified. Their implementation and use led to the relaxation of certain auction rules, including non-binding bids and non-binding auctions, the latter often being a hybrid, i.e., auction-followed-by-negotiation mechanism. Below is a discussion on the benefits and weaknesses of these auctions followed by a brief review of software implementation and real-life examples.

2.1 Multi-attribute auctions

In procurement, price-only auctions have been used as a competitive, transparent and accountable allocation mechanism. However, a common criticism has been that price-only auctions focus solely on one attribute, while other, no less important attributes are ignored. Growing pressures to seek value for money led buyers to introduce mechanisms which allow for the assessment of prices as well as qualities of proposals in the whole life-cycle of a product.

Multi-attribute auctions extend price-only auctions and allow bid-takers to incorporate various characteristics of a product or service that are important to them (Rothkopf and Whinston, 2007). Che (1993) and Branco (1997) formulated early models of two-attribute (price and quality) auctions in a procurement context. Suppliers could formulate two-attribute bids because they were informed about the buyers' preferences in terms of a scoring function. Also, the suppliers' costs associated with quality depended on a single parameter. This made it possible to apply the single-attribute auction design apparatus to multi-attribute cases. Shachat and Swarthout (2010) modified the model for procurement in which every supplier produces goods of the same quality. The buyer knows the different qualities and prior to the auction she awards suppliers with credits basing it on the differences in quality. Vulkan and Jennings (2000) extended Che's protocols to multiple non-price attributes. They proposed a multi-attribute auction in which the buyer provided suppliers with a utility function, the maximum amount she was willing to pay, and the minimum level of utility required.

The requirement that the buyer discloses her utility is weakened by the requirement that a distorted function be disclosed. The distorted function has to be of a particular form in order to assure the winning bid's efficiency (David, Azoulay-Schwartz et al., 2006). David et al. (2006) designed mechanisms for a first-score sealed bid, second-score sealed bid, and English auctions with full disclosure of distorted information. In all these models private information is represented by a single attribute, while in procurement there may be more than one attribute (Perrone et al., 2010).

These auctions were designed to study the bid-makers' optimal strategies and to solve winner

determination problems when suppliers obtained full, partial or distorted information about the buyer's preferences. The model proposed by Karakaya and Köksalan (2011) was designed to support the suppliers' bidding decisions based on prior knowledge of the buyer's past preferences; after each round the model uses information about the winning bid to update the estimation of the bid-taker's preference values.

2.2   Non-binding bids and non-binding auctions

There are complexities associated with setting multi-attribute auctions as compared to price-only auctions. There is also a requirement that the buyers disclose their utility or any other function, which is used to evaluate bids to the suppliers. This may be an acceptable requirement for auctions conducted by different governmental organizations—for example, the European Union has recently adopted a new public procurement directive, which requires that organizations disclose measures they use to assess bids. However, businesses prefer to keep their measures secret because disclosure may reveal their strategies and trade secrets (Burmeister et al., 2002). Lack of common acceptance of disclosure led to the design of other exchange mechanisms and to the modification of auctions. For example, some modifications allow making non-binding bids; a bidder may decline to sign a contract despite winning an auction. Cramton and Katzman (2010) observe, however, that such auctions undermine the credibility of bids and encourage low-ball bids.

There are also non-binding auctions (so-called buyer-determined), which became popular in procurement. An exploratory survey of buyers from four *Fortune*-100 businesses shows that these organizations often used non-binding auctions (Jap, 2002). This two-stage mechanism aims to address the difficult problem of multi-attribute goods. In the first stage, a price-only reverse auction is conducted. In the second stage, the buyer, following a successful price-only auction, invites one or a few suppliers (including those who were not the auction's participants) to move to a negotiation phase and negotiate the contract (Engelbrecht-Wiggans et al., 2007). Not only may the auction winner not get the contract but she may not be invited to the bargaining table. Therefore, participants of non-binding reverse auctions find buyers opportunistic and the process ambiguous and opaque (Jap, 2002; Gupta et al., 2012).

Auctions with non-binding bids and non-binding auctions are quasi-auctions because they do not include one of the key ruled of auctions, which is winner determination. They attempt to address the drawback of the price-only reverse auction focus, which is "driving the price down". This often comes at the cost of non-price attributes, which are very important yet not included in the process. The purpose of multi-attribute auctions is to include both price and non-price attributes so that there is no need for mechanisms which augment auctions with negotiations. However, the wide spread use of no-biding auctions followed by negotiations in procurement indicates practical limitations of multi-attribute auction mechanisms.

2.3   Business solutions

When multiple attributes are replaced with one (price) attribute, then there is a loss of the auctions' efficiency (Bichler, 2000; Asker and Cantillon, 2008). This loss may increase when the price-only auction is followed by a post-auction negotiation during which price may be re-negotiated. There are, however, approaches which, depending on the specific buyer's circumstances (e.g., risk aversion and the shape of the utility) propose to select either a multi-attribute auction or a negotiation (Huang et al., 2013).

Other drawbacks of non-binding bids and non-binding auctions (e.g., perceived opportunism,

exchange opacity, and process inefficiency) motivated the development of software that would be robust and easy to use. The so-called, e-sourcing engines, were developed so that buyers could use price as well as "a weighted scoring auction, discounting these factors, against the responses by your suppliers, giving you a system calculated comparative analysis of the bids, same as your cost comparison sheets." (Mane, 2012).

These capabilities have been offered by many vendors, including, SAP's Sourcing application, CombineNet's Advanced E-sourcing application, Oracle's Sourcing application, and PeopleSoft's Strategic Sourcing. The examples posted by these vendors show that scoring was equated with costing so that every multi-attribute alternative was measured with money. These and other procurement software developers incorporated scoring capability in their auction designs. The key problem that has been resolved in different ways is the allocation of knowledge of scoring functions. One option is to give the buyer's scoring function to all suppliers (Koppius and van Heck, 2003). Another option, patented by Ariba, Inc. (Wellman, 2005), is to give all scoring functions to the "auction engine" which matches buyers' and suppliers' bids by decomposing every submitted bid-price into its attribute components. Scoring has also been implemented by Epicor Software, IBM/Digital Union, Perfect Commerce, TradeExtensions, and a number of other software platforms; Bausa Peris et al. (2013) report that in EU 11 platforms used in public procurement use scoring functions.

Scoring functions, which have been gaining popularity in the private and public sectors, do not enforce the quasi-linearity assumption. They are often piece-wise linear with bounds imposed on the attribute values. They are not expressed in terms of money but score, which is not transferable.

One could argue that, while there may be room for improvement, scoring functions work sufficiently well and their use yields results that satisfy buyers. There are however, at least three issues that may undermine this kind of argument: (1) while theory requires that suppliers use cost-based scores, in practice they may use profit- or value-functions; (2) price is assumed to be a linear function but in many cases buyers use pricing models which incorporate partial or even total cost of ownership (Plank and Ferrin, 2002), which is unlikely to be linear due the likelihood of a breakdown, return, repair, and discharge; and (3) although the implemented solutions may work well for the buyers they may cause market inefficiency and needlessly weaken suppliers. While this last issue is the main concern here, Plank and Ferrin's work also adds to the discussion on the first two problems. The measures which incorporate preferences of market participants and which they use to compare the alternatives lie at the root of these problems.

2.4 Examples of real-life multi-attribute auctions

Literature review indicates that real-life examples of quasi-linear preferences in bidding and bargaining situations are rare. It is also difficult to find an analysis of the preferential structure of multi-attribute decision problems in procurement.

Mars, Inc. created an electronic private exchange in which volume discount bidding and multi-attribute bidding were used most often (Hohner et al., 2003; Bichler et al., 2006). The attributes included payment and its terms (e.g., pre-payment, payment date, and discount) as well as turnaround time, delivery schedule, product quality, type of material, and color. Bichler et al. (op. cit.) report that one of the reasons why Mars chose iterative auctions was their similarity to negotiations–the process that both suppliers and procurement managers were familiar with. An important difference between the two mechanisms is that auctions assume quasi-linearity while negotiations do not. Irrespective of the specifics of the auctions and its attributes (due to the non-disclosure agreement the authors could not describe the case in detail), one may ask if the

auctions' participants knew about this assumption and the implications of its violation. It is likely that the payment and the terms of payment are interdependent. In such a situation they should be incorporated into a single attribute payment. If however, pre-payment, discount and other terms were subject to bidding, they could not be combined with the payment attribute. In effect the scoring function was not quasi-linear.

The Mars-IBM procurement auction platform was created in 2001 and it operated as a subsidiary (Freight Traders) of Mars until 2006 when most employees joined Trade Extensions (TradeExtensions.com). Trade Extensions created a procurement software platform which includes reverse auctions and negotiations. A review of four procurement case studies (i.e., Ineos, Road resurfacing, Elderly Care Services, and Cleaning Services available from http://www.tradeextensions.com/case_studies.asp) shows that Trade Extension's auction mechanism requires that all attributes and, in the case of combinatorial auction, packages must be expressed in monetary terms. The focus is on the minimization of the costs of procured services subject to constraints imposed on the attributes and packages. There is no indication that the costs and revenue functions need to be quasi-linear; in fact, in most cases all attributes are discrete. Another feature in all cases is that the bidding concerns services or goods that would be produced in the future and would make price and other attributes interdependent.

The European Union has recently adopted a new public procurement directive; it requires that the procurement authority publish ex ante relative weighting of each criterion. The E.U. directives (Article 55 in 2004/17/EC or Article 53 in 2004/18/EC) require that public contracts be allocated by competitive bidding. The buyer has to either use a scoring function in which price and other attributes and their weights are given or a lexicographically ordered list of attributes. Lundberg and Marklund (2011) argue that a multi-attribute scoring function should be used because it can represent society's preferences. This may be the case, but the society's preferences related to a single transaction are likely to be in conflict with the society's preferences related to the functioning of the national or regional economy. Lundberg and Marklund (op. cit., p. 66) also note that the representation of the buyer's utility with a quasi-linear function is reasonable because "Commonly, the price of the procured product constitutes only a small fraction of the procuring authority's total budget." Neither the directive nor the authors address the issue of the utility of the suppliers and situations in which the expense is a significant item in the municipal and other public organizations' budget.

In the U.S., scoring auctions, known as "A+B bidding", have been used for the procurement of highway construction work. (Gupta, Snir et al., 2013). The highway authorities evaluate offers on the basis of: (1) the total costs (A); and (2) the number of days required to complete the project (B) weighted by a road user cost (i.e., the difference between the road user costs during construction and the costs after construction is completed). Asker and Cantillon (2008, p. 70) report that by 2003, 38 U.S. states were using auctions with scoring functions for large projects for which time was a critical factor. The state's transportation authority utility is assumed to be quasi-linear; $u_b(A, B) = A + d{\times}B$ (where d is the road user cost parameter). This assumption appears false; although A and B are separable, they are interdependent, e.g., reduction of time required to complete the project may be achieved by adding more workers and equipment to the project, i.e., increasing its costs (El-Rayes and Kandil, (2005). Furthermore, both completion time and the project's costs are typically discrete attributes and the trade-off function between them is non-linear (i.e., cost increase does not decrease completion time uniformly and vice versa).

Auctions tend to focus on the owner (buyer in procurement); as long as participation in an auction can be assured, the mechanism should "take care of the process". It needs to be carefully designed so that "the auction mechanism implements a solution that maximizes the total payoff across all

agents" (Bichler and Kalagnanam, 2006, p. 106). While this is a well-defined objective, its implementation may not be clear to auction creators and their users. Even if the buyer's utility quasi-linearity is verified, there is no guarantee that every participating supplier's utility is also quasi-linear. If it is not, then there are no assurances for meeting the stated objective.

Multi-attribute auctions are difficult for suppliers who need to be able to construct their own scoring functions and form bids on the basis of their own and the buyer's scorings. The number of attributes and the relationships among them may introduce a level of complexity which sellers cannot manage effectively. Consider a series of multi-attribute auctions, which began in 1999 and in which over 50 health plan providers (i.e., suppliers) competed for business from three large employers (i.e., buyers: IBM, Morgan Stanley, and Ikon Office Solutions). The 1999 auctions reduced annual rates between 2 and 8% ($1.1 million) as compared to other employers' increases of between 4 and 6%, and the negotiation time was reduced from five weeks to one week (Gupta, Parente et al., 2012, p. 305). The 1999 success of the auctions led to their expansion in 2000 from 50 to 100 health providers and nine employers. In 2001, however, only four out of nine employers were participating in the auctions, subsequently the auctions were discontinued (op. cit., p. 305).

Gupta, Parente and Sanyal (2012) seek the roots of the failure of these auctions in the design flaws of the mechanism's information feedback and winner determination. The auctions were only partially transparent—suppliers did not know the weight of the non-price attributes and which values they were required to submit. Most bidders did not revise their bids even though all bids were disclosed because they did not know how winners would be chosen (op. cit., p. 316). Another flaw of these auctions was the treatment of attributes. There were four price attributes and a number of non-price attributes, some of which were interdependent with the price attributes. The price attributes were aggregated into a composite score. This score did not take into account attributes associated with service costs (e.g., safety, quality of service, and response time), which affected each of the four prices and to a different extent.

Although the focus here is on multi-attribute reverse auctions, this section concludes with an example of a single attribute reverse auction because it refers to the bidders' utilities. Procaccia (2013) observes that some houses in Pittsburg are sold via first-price sealed-bid auction. He considered introducing an online second-price auction but noted that bidders' preferences are not quasi-linear because the price that a bidder offers for a given house does not reflect the subjective value of the house. Instead it reflects the two contradictory forces at play: one is the bidder's conviction that this house or another house of similar value could be bought in future for less than $x, the second force is the anxiety that the bidder may become homeless, which would be much worse than buying the house for $x.

3. Quasi-linear utilities, efficient solutions, and social welfare

A discussion about the implications of quasi-linear utility gives an insight into the role of constraints and the loss of efficiency of the mechanism and/or its solution when utilities are not quasi-linear.

3.1 Quasi-linear utility

Let x be a vector of attributes of a good (service) $x = [x_1, ..., x_N] \in X$. In general, the attributes may be real numbers, ordinal, categorical, and nominal. In this section attribute x_1 is assumed to be a real number and the remaining attributes $x_2, ..., x_N$ are assumed to be non-negative real numbers, which is an often made simplification. For simplicity, we assume that every attribute x_i is bounded from above, i.e., $X = \{0 \leq x \leq x^*\}$.

The first attribute x_1 is numeraire and, following the convention, it is assumed to be price. Vector $\mathbf{x}_{-1} = [x_{2, ..., } x_N] \in X^{-1}$ is a vector of all attributes except for price; $\mathbf{x}_{-1}$ is a configuration of the good (service), which is the subject of the transaction. Set X^{-1} is the set of all feasible configurations.

The quasi-linear utility function of the buyer u_b can be formulated as follows:

$$u_b(\mathbf{x}) = v_b(\mathbf{x}_{-1}) - x_1, \tag{1}$$

where: $\mathbf{x}_{-1}$ ($\mathbf{x}_{-1} \in X^{-1}$) is the good, $v_b(\mathbf{x}_{-1})$ is the valuation function of the good, which is strictly concave (twice differentiable with $v_b' > 0$; $v_b'' < 0$, and bounded from above), and x_1 is the price.

Correspondingly, the seller's i ($i \in I$) utility function is:

$$u_i(\mathbf{x}) = x_1 - v_i(\mathbf{x}_{-1}), \tag{2}$$

where: $v_i(\mathbf{x}_{-1})$ is the valuation function of seller i (often assumed as costs), which is convex (twice differentiable with $v_i' > 0$; $v_i'' \geq 0$,).

The indifference curves of quasi-linear utilities (1)-(2) are convex functions in X^N. If the numeraire is positioned on the horizontal axis, then the indifference curves are horizontal translates of each other as illustrated in Figure 1 for $N = 2$.
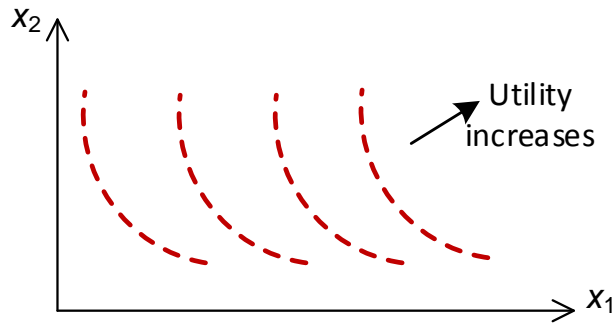

Figure 1. Indifference curves for a quasi-linear utility

The marginal utility of price is constant ($\partial u(\mathbf{x})/\partial x_1 = 1$) and it does not depend on any attribute value of the product other than the price. Correspondingly, the marginal utility of the nonlinear component does not depend on the price, that is: $\partial u_b(\mathbf{x})/\partial \mathbf{x}_{-1} = \partial v_b(\mathbf{x}_{-1})/\partial \mathbf{x}_{-1}$ (for the buyer) and $\partial u_i(\mathbf{x})/\partial \mathbf{x}_{-1} = \partial v_i(\mathbf{x}_{-1})/\partial \mathbf{x}_{-1}$ (for the seller). This means that buyer's b valuation of good $\mathbf{x}_{-1}$ does not depend on the money which she has to pay. Similarly, seller's i ($i \in I$) valuation (costs) does not depend on the money he receives upon selling the good.

3.2 Efficient solutions

An exchange process can be represented using the Edgeworth box. The case when both sides have quasi-linear preferences is illustrated in Figure 2. Because for both the buyer and the seller the indifference curves are horizontal translates of each other, *the contract curve is an interval*, i.e., all efficient solutions lie on the horizontal (dotted) line. Efficient solutions differ in the value of price (x_1), indicated in Figure 2 as a sequence of numbers (15, 40, ...). The value of the second variable x_2, which describes the non-price attribute of the exchanged good is the same in every efficient solution; it is $x_2(s)$ for the seller and $x_2(b)$ for the buyer, and $x_2(s) = x_2(b)$.
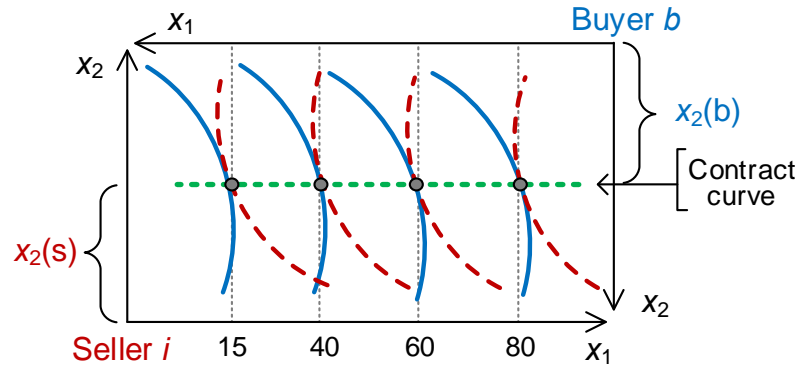
Figure 2. Efficient solutions in exchanges under quasi-linear preferences

Strecker (2010, p. 274) observed that given the buyer's valuation and the seller's costs, there is only one good configuration for every seller. The reverse is also true, if seller i ($i \in I$) can propose different efficient configurations, then at least one of the utilities $u_b(\boldsymbol{x})$ and $u_i(\boldsymbol{x})$ is not quasi-linear.

A configuration of the good is represented by the vector of attribute values $\boldsymbol{x}_{-1} \in X^{-1}$. The above discussion suggests that there is only one feasible configuration that can be considered in an efficient exchange between buyer b and seller i ($i \in I$).

<u>Proposition 1</u>. Utilities $u_b(\boldsymbol{x})$ and $u_i(\boldsymbol{x})$ ($i \in I$) are quasi-linear functions, defined by (1) and (2), respectively, and set $\tilde{X}_i$ is the set of efficient solutions representing trades between buyer b and seller i, if and only if there is only a single configuration of efficient attribute values $\tilde{\boldsymbol{x}}_{-1}$.

<u>Proof: Part A (by contradiction)</u>: Let's assume that there are two different configurations of efficient attribute values: $\tilde{\boldsymbol{x}}_{-1}$ and $\dot{\boldsymbol{x}}_{-1}$. That is $\tilde{\boldsymbol{x}} = (\tilde{\boldsymbol{x}}_{-1}, \tilde{x}_1)$ and $\dot{\boldsymbol{x}} = (\dot{\boldsymbol{x}}_{-1}, \dot{x}_1)$ are efficient solutions and $\tilde{\boldsymbol{x}}_{-1} \neq \dot{\boldsymbol{x}}_{-1}$.

We define two families of indifference curves tangential at the same point (indicated by index j):

$$U_{bj}: u_{bj}(\boldsymbol{x}) = v_b(\boldsymbol{x}_{-1}) - x_1 = u_{bj} \tag{3}$$

and

$$U_{ij}: u_{ij}(\boldsymbol{x}) = x_1 - v_i(\boldsymbol{x}_{-1}) = u_{ij}. \tag{4}$$

Let's assume that curves U_{b1} and U_{i1} are tangential at $\tilde{\boldsymbol{x}} = (\tilde{\boldsymbol{x}}_{-1}, \tilde{x}_1)$, while U_{b2} and U_{i2} are tangential at $\dot{\boldsymbol{x}} = (\dot{\boldsymbol{x}}_{-1}, \dot{x}_1)$. Let

$$d = \tilde{x}_1 - \dot{x}_1. \tag{5}$$

Given that two quasi-linear utilities' indifference curves u_{b1} and u_{i1} are tangential at $(\tilde{\boldsymbol{x}}_{-1}, \tilde{x}_1)$, from (3) and (4) we obtain that the following two indifference curves are tangential at $(\tilde{\boldsymbol{x}}_{-1}, \tilde{x}_1 + d)$:

$$U_{b3}: u_{b3}(\boldsymbol{x}) = v_b(\boldsymbol{x}_{-1}) - (x_1 + d) = u_{b1} - d \tag{6}$$

and

$$U_{i3}: u_{i3}(\boldsymbol{x}) = (x_1 + d) - v_i(\boldsymbol{x}_{-1}) = u_{i1} + d. \tag{7}$$

Indifference curves U_{b3} and U_{i3} given respectively by (6) and (7) differ in the value of the numeraire; they are a translation along dimension x_1 (see Figure 1).

From (5) it follows that for the tangential points of pairs (U_{b2} and U_{i2}) and (U_{b3} and U_{i3}), the variable x_1 takes the same value $\dot{x}_1 = \tilde{x}_1 - d$. Therefore, if tangential point $(\dot{\boldsymbol{x}}_{-1}, \dot{x}_1)$ of pair (U_{b2} and U_{i2}) is

different from tangential point $(\widetilde{\boldsymbol{x}}_{-1}, \widetilde{x}_1)$ of pair $(U_{b3}$ and $U_{i3})$, the difference has to be in their respective components $v_b(\boldsymbol{x}_{-1})$ and $v_s(\boldsymbol{x}_{-1})$. This requires that $v_b(\boldsymbol{x}_{-1})$ and $v_s(\boldsymbol{x}_{-1})$ be tangential at two different points $\dot{\boldsymbol{x}}_{-1}$ and $\widetilde{\boldsymbol{x}}_{-1}$, which is not possible because by definition they are concave and convex, respectively. Hence, $\widetilde{\boldsymbol{x}}_{-1} = \dot{\boldsymbol{x}}_{-1}$.

<u>Part B.</u> We assume that $|X^{-1}| > 1$, i.e., there are at least two different feasible configurations and x_1 is a real number. If two configurations are efficient, then $u_b(\boldsymbol{x})$ and $u_i(\boldsymbol{x})$, $(i \in I)$, are not quasi-linear functions. If only one configuration is efficient, then it is efficient for different values of x_1. This means that the marginal utility of price is constant and the marginal utility of the good configuration does not depend on price. From the fact that there is only one efficient configuration it follows that $u_b(\boldsymbol{x})$ and $u_i(\boldsymbol{x})$ are concave and convex, respectively, and tangential at the efficient configuration. From the fact that efficient solutions differ in price only, it follows that the contract curve is an interval as shown in Figure 2. ♦

The valuation of good $\widetilde{\boldsymbol{x}}_{-1}$ by buyer b is $\tilde{v}_b = v_b(\widetilde{\boldsymbol{x}}_{-1})$, while seller's i valuation is $\tilde{v}_i = v_i(\widetilde{\boldsymbol{x}}_{-1})$. From Proposition 1 it follows that every configuration $\boldsymbol{x}_{-1}$, other than $\widetilde{\boldsymbol{x}}_{-1}$, that enters an exchange between buyer b and seller i is inefficient.

The valuation of good $\widetilde{\boldsymbol{x}}_{-1}$ buyer b is $\tilde{v}_b = v_b(\widetilde{\boldsymbol{x}}_{-1})$, while supplier's i valuation is $\tilde{v}_i = v_i(\widetilde{\boldsymbol{x}}_{-1})$. From Proposition 1 it follows that every other good described by the attributes $\boldsymbol{x}_{-1}$ that enters an exchange between buyer b and supplier i would be inefficient. Because the good's valuations are nonnegative and dominated by a single point $\widetilde{\boldsymbol{v}} = [\tilde{v}_b, \tilde{v}_{i*}]$ every set of feasible valuations is contained in set (rectangle) C^{bi} shown in Figure 3.
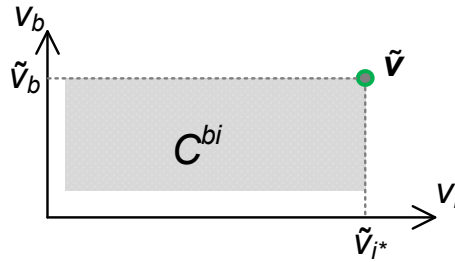

Figure 3. Space of the good's valuations by buyer b and supplier i

Since there is a unique efficient solution $\widetilde{\boldsymbol{v}} = [\tilde{v}_b, \tilde{v}_i]$, this solution maximizes the value of any convex and linear functions $f(v_b, v_i)$, which includes the sum of valuations, i.e., $f(v_b, v_i) = v_b + v_i$.

3.3 Social welfare and efficiency

Social welfare or total value allocation is defined in auction theory as the sum of buyer's b utility (1) and the utility (2) of seller i who obtained the contract (Krishna, 2009; Strecker, 2010)[1], that is:

$$u_b(\boldsymbol{x}) + u_i(\boldsymbol{x}) = v_b(\boldsymbol{x}_{-1}) - x_1 + x_1 - v_i(\boldsymbol{x}_{-1}) = v_b(\boldsymbol{x}_{-1}) - v_i(\boldsymbol{x}_{-1}) \qquad (8)$$

---

[1] The use of Nash solution, i.e., the product of the utilities when there are no status quo values, may be preferable because of its independence from affine utility transformation in terms of value allocation. However, the results would be less elegant because the price would continue to affect the value allocation. Instead, the sum of utilities allows us to consider price as internal payment transfer between the buyer and the supplier, with no impact on social welfare.

Proposition 1 states that in an exchange between b and i ($i \in I$) there is only one efficient configuration and efficient solutions may differ in price only. Let's assume that $\underline{x}_{1i} \geq 0$ is the lowest price at which seller i can sell $\widetilde{x}_{-1}$. From (1) and (2) we obtain that buyer's b maximum utility (surplus) is $\tilde{u}_b = v_b(\widetilde{x}_{-1}) - \underline{x}_{1i}$ and for seller i the minimum utility is $\tilde{u}_i = \underline{x}_{1i} - v_i(\widetilde{x}_{-1})$. Any increase in $\underline{x}_{1i}$ decreases the buyer's surplus and increases the seller's surplus. Points $\widetilde{v} = [\tilde{v}_b, \tilde{v}_i]$ and $\widetilde{u} = [\tilde{u}_b, \tilde{u}_i] = [\tilde{v}_b - \underline{x}_{1i}, \underline{x}_{1i} - \tilde{v}_i]$, are shown in Figure 4.
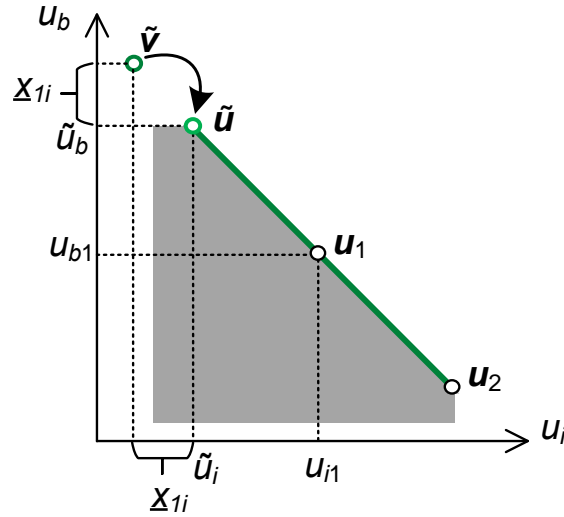

Figure 4. Efficient frontier in buyer's b and seller's i utility space

The shift from the maximum good valuation $\widetilde{v}$ to maximum social welfare $\widetilde{u}$ is the result of transfer of price $\underline{x}_1$ from b to i. Social welfare does not change because every increase in the seller's utility is matched by an equivalent decrease of the buyer's utility and vice versa. This means that the utility possibility frontier (efficient frontier) is an interval $[\widetilde{u}, u_2]$, which originates from $\widetilde{u} = [\tilde{u}_b, \tilde{u}_i]$ as it is shown in Figure 3. Point u_2 corresponds to the situations in which buyer b pays seller i the maximum possible price. From (8) it follows that solutions represented by points on the interval $[\widetilde{u}, u_2]$ yield the same value of social welfare.

Assuming that attribute values are non-negative and bounded from above and there are no other constraints on the attribute configuration, the auction winner i^* makes bid x^*, which is the solution of the following problem:

$$(i^*, x^*) = \arg\max_{i \in I, x \in X}\big(u_b(x) + u_i(x)\big) \tag{9}$$

We assume that a solution of (9) exists and there is only one auction winner i^*. If (9) has multiple solutions for different values of i, then one pair (i^*, x^*) is arbitrarily selected.

According to Proposition 1, there is a configuration x^*_{-1} offered by winner i^*, which is a part of the solution (9) and which maximizes social welfare u_{bi}, i.e.,

$$\tilde{u}_{bi*} = \max_{x \in X}\big(u_b(x) + u_i(x)\big) = v_b(x^*_{-1}) - v_{i*}(x^*_{-1}). \tag{10}$$

The maximum social welfare is obtained when good x^*_{-1} is selected. However, a reverse auction may not end up with a winner when neither the buyer nor the seller accept a negative surplus. Therefore, we require that:

$$v_b(\boldsymbol{x}_{-1}) - x_1 \geq 0 \tag{11}$$

and

$$x_1 - v_i(\boldsymbol{x}_{-1}) \geq 0 \; (i \in I). \tag{12}$$

Condition (11) may be set by the buyer as the reservation price. Condition (12) means that sellers cannot incur losses, i.e., price x_1 cannot be lower than the costs of producing good $\boldsymbol{x}_{-1}$. Observe, that these two conditions taken together require that buyer's valuation has to be greater than the winning seller's costs, i.e., $\tilde{u}_{bi*} \geq 0$. Thus, if a solution to (9) exists but $\tilde{u}_{bi*} < 0$, then the auction does not produce the winner.

From (1) and (12) it follows that buyer b achieves maximum surplus when the price equals the winning seller's costs, i.e.,

$$x_1^* = v_{i*}(\boldsymbol{x}_{-1}^*). \tag{13}$$

3.4   Configuration and price bidding

In price-only auctions there is only one configuration for all sellers. Therefore, the buyer's valuation $v_b(\boldsymbol{x}_{-1})$ is a constant. The seller, who can offer good $\boldsymbol{x}_{-1}$ at the lowest price, wins. The process is somewhat more complicated in multi-attribute auctions.

According to Proposition 1, for every seller i ($i \in I$) there is one efficient configuration $\boldsymbol{x}_{-1,i}^*$, i.e., there are $|I|$ efficient configurations. If the buyer has unlimited budget (price is irrelevant), then the seller who offers the configuration which maximizes buyer's b valuation wins the auction. If price matters, then the seller who maximizes the buyer's utility (1) wins the auction. Although we assumed that (9) has a unique solution, in general there may be multiple efficient solutions offered by different sellers. These solutions differ in configuration and price; one seller may bid a less valued configuration at a lower price and another seller may bid a more valued configuration at a higher price. Both bids yield the same utility for the buyer.

The sellers' ability to modify configurations lead, in light of Proposition 1, to a practical difficulty: How to determine an efficient configuration through bidding? The sellers may assume that the buyer prefers to pay less but they do not know the buyer's valuation of different configurations $\boldsymbol{x}_{-1}$ ($\boldsymbol{x}_{-1} \in X^{-1}$). In order to conduct a multi-attribute auction some information about the buyer's preferences must be conveyed to the seller. One possibility is that the sellers obtain information that gives them clear indication regarding their efficient configurations. In such a case the multi-attribute auction can be replaced with a single-attribute auction.

<u>Proposition 2</u>. If problem (9) has a unique solution, the buyer's and the sellers' utilities are quasi-linear and the sellers know their efficient configuration, then a multi-attribute auction is equivalent to a single-attribute auction.

<u>Proof</u>. Given that (9) has a unique solution and the utilities are quasi-linear, we obtain:

$$\forall i \; (i \in I) \; \exists \, \boldsymbol{x}_{-1,i}^*: \; \tilde{u}_{bi}\big(\boldsymbol{x}_{-1,i}^*\big) > \tilde{u}_{bi}\big(\boldsymbol{x}_{-1,i}\big) \; and \; v_b\big(\boldsymbol{x}_{-1,i}^*\big) > v_b\big(\boldsymbol{x}_{-1,i}\big),$$

where: $\boldsymbol{x}_{-1,i}^*$ is seller's i efficient configuration, $\boldsymbol{x}_{-1,i} \in X^{-1,i}$, $X^{-1,i}$ is the set of feasible configurations for i.

If seller i wants to win the auction, she has to offer buyer b configuration $\boldsymbol{x}_{-1,i}^*$. Thus, for every seller i ($i \in I$), the configuration and its valuation $v_b(\boldsymbol{x}_{-1,i}^*)$ are constant during the auction. Seller i bids only on price x_{1i} which is used to obtain buyer's b utility $u_b(x_{1i}) = v_b(\boldsymbol{x}_{-1,i}^*) - x_{1i}$. The auction's winner is seller i^* given by (13); i^* offers the highest value of the difference between the

constant $v_b(\boldsymbol{x}^*_{-1,i*})$ and price $\boldsymbol{x}^*_{1,i}$. ♦

Proposition 2 generalizes Lemma 1, formulated by David et al. (2006, p. 534), that the sellers can select configuration that maximizes their utilities independently of price providing they are given the buyer's valuation function. Proposition 2 states that because every seller has only one efficient configuration selection (maximizing social welfare and the buyer's valuation) the multi-attribute auction can be replaced with price-only auction in which individual bids are discounted to account for the buyer's valuation of the configurations.

During the auction, the sellers do not have to know the buyer's valuation of their efficient configuration. For every bid x_{1i} the mechanism computes the buyer's utility $u_b(x_{1i})$ and reveals the winning bidder. Other bidders can stay in the auction only if they offer price levels that increase the buyer's utility over the winning bid's utility. Thus, the price decrease has to account for the difference in the buyer's valuation of the sellers' configurations. Therefore, the seller who offers the lowest price may not win the auction.

4. Limitations of quasi-linear utilities

Quasi-linear preferences have been employed by economists in order to study competitive markets. The independence of price and valuation as well as the requirement that the valuation functions are twice differentiable and always increasing help to theorize about firms' production and exchange decisions. However, these and also other assumptions underlying quasi-linear preferences are difficult to meet in most procurement auctions. In this section different sources for quasi-linearity violation are discussed and examples given.

4.1 Price, budget, and ownership cost

Quasi-linear preferences require that the market participants be risk-neutral. If they are risk averse (seeking), then their utility is concave (convex) and the price cannot be separated from valuation (Krishna, 2009, pp. 38-42). The assumption that participants are risk neutral is often unrealistic; risk aversion has been used to explain overbidding behavior (Bajari and Hortacsu, 2005). In procurement auctions, sellers of timber and construction firms were found to be risk averse (Athey and Levin, 2001; Campo, 2012). Procurement managers in public organizations were found more risk averse than their counterparts in private organizations (Boyne, 2002). (The risk neutrality assumption is required in both price-only and multi-attribute auctions.)

Another problem may arise due to constraints imposed on the attribute levels. Budget and other constraints are often necessary in procurement, however, their effect is that no efficient and strategy proof mechanism that meets these constraints can be constructed (Hurwicz and Walker, 1990).

Quasi-linear preference assumption requires that the buyers' valuation functions be strictly concave (twice differentiable, and always increasing at a decreasing rate) and the sellers' costs functions must be convex (having non-decreasing marginal costs). A review of procurement guides suggests that government buyers use linear or piece-wise linear cost functions.[2] The costs range from costs equal to price to the total costs of acquisition (TCA) to the total cost of ownership

---

[2] See, for example: http://www.business.govt.nz/procurement/pdf-library/agencies/Guidetomeasuringprocurementsavings.pdf; http://www.netl.doe.gov/energy-analyses/pubs/QgESSNETLcostEstmethod.pdf; https://buyandsell.gc.ca/policy-and-guidelines/supply-manual/section/10; http://gsd.mt.gov/content/procurement/RFP_Process/Cost_Evaluation.pdf.

(TCO); both TCA and TCO may include a large number of factors (Ferrin and Plank, 2002). The use of TCA, TCO and similar methods causes that the buyer's utility function becomes a sum of different cost types:

$$u_b(\boldsymbol{x}) = v_b(\boldsymbol{x}_{-1}) - \sum_{l=1}^{L} c_{lb}(\boldsymbol{x}_{-1}) - x_1, \tag{14}$$

where $c_{lb}(\boldsymbol{x}_{-1})$ is the l-th type of cost and $C_b(\boldsymbol{x}) = \sum_{l=1}^{L} c_{lb}(\boldsymbol{x}_{-1}) + x_1$ is the total cost.

If all types of costs are included in the numeraire then the buyer's and the sellers' numeraire are incompatible. If the buyer's valuation functions is $v_b(\boldsymbol{x}_{-1}) - \sum_{l=1}^{L} c_{lb}(\boldsymbol{x}_{-1})$, then it may not be a concave function when $c_{lb}(\boldsymbol{x}_{-1})$ are quasi-linear functions. A change of some attribute levels (e.g., payment and color) may have little or no impact on total costs, while a change of other attribute levels (e.g., quality and warranty) may cause a jump in total costs.

4.2 Efficient configuration and the buyer's surplus

The requirement that the buyer's valuation of the good be greater than the seller's costs causes that the quasi-linear assumption may not be sufficient to assure allocative efficiency, solution efficiency, and surplus maximization. Another requirement is that there be configurations $\boldsymbol{x}_{-1} \in X^{-1}$, such that functions $v_b(\boldsymbol{x})$ and $v_i(\boldsymbol{x})$ are tangential at $\boldsymbol{x}_{-1}$. If this requirement is not met, then efficient configurations are on the boundary of X^{-1} (Kersten and Noronha, 1998). In such a situation, a change of the constraints imposed on attribute level changes efficient frontier (until tangential points are reached).

One condition for $u_b(\boldsymbol{x})$ and $u_i(\boldsymbol{x})$ to be tangential at $\boldsymbol{x} \in X$ is that the buyer's preferences should be opposite to those of the sellers (i.e., if the buyer prefers a higher level of an attribute, then the sellers prefer a lower level). If there is an attribute, increase of which is preferred by both the buyer and the seller, then both sides want this attribute to reach infinity or a boundary Level. The example discussed by Chen-Ritzo et al. (2005a; 2005b) can be used to illustrate this situation. This example is modified here in order to illustrate the case when the winning configuration does not maximize the buyer's surplus.

<u>Example 1</u>. The buyer's valuation is: $v_b = 16.5\ x_2^{0.4} + 33(10 - 1.3\ x_3^{0.1})$, where x_2 and x_3 are the attributes describing quality and lead time. Seller's 1 valuation is: $v_1 = 0.7\ x_2^{1.7} + 5(16 - x_3^{0.9})$; and Seller's 2 valuation is: $v_2 = 6.5(16 - x_2^{0.2}) + 0.5\ x_3^{1.95}$. (Note: the parameters of the valuation functions are modified here in order to disallow negative valuations and to differentiate between the two sellers.) The feasible set of alternatives is $X^{-1} = \{x_2=1, 2, ..., 10; x_3=1, 2, ..., 10\}$.

Values of different configurations in the buyer's and two sellers' ($i = 1, 2$) valuation space $V = \{\boldsymbol{v}_{kl} = [v_k^b, v_l^i], k, l = 1, 2, ...\}$ are shown in Figure 4; note that every feasible configuration appears twice, separately for each seller. The best configuration for the buyer is $\boldsymbol{x}_{-1}^* = [10; 1]$; its valuation is $v_b(\boldsymbol{x}_{-1}^*) = v_b(10, 1) = 338$. The costs of this configuration is 110 for Seller 1 and 94 for Seller 2. If Seller's 2 bid on price is 94, then the buyer's surplus (and social welfare) is $u_b(10, 1, 94) = 244$.

Configuration $\boldsymbol{x}_{-1} = [10; 10]$ yields valuation $\boldsymbol{v}_{12} = [v_b(10, 10); v_1(10, 10)] = [75, 330]$ when Seller 1 offers it and $\boldsymbol{v}_{22} = [138, 330]$ when it is offered by Seller 2. This configuration is worse for the buyer than [10; 1]. Seller's 1 cost of producing this configuration is 75 and for Seller 2 the cost is 138. Seller 1 can offer price 85 so that the buyer's surplus is $u_b(10, 10, 85) = 245$ and it is greater than the highest buyer's surplus that Seller 2 can offer (i.e., $u_b(10, 1, 110) = 244$). Social welfare produced by $\boldsymbol{v}_{12}$ is 255 (see Figure 5) and Seller's 1 surplus is 10.
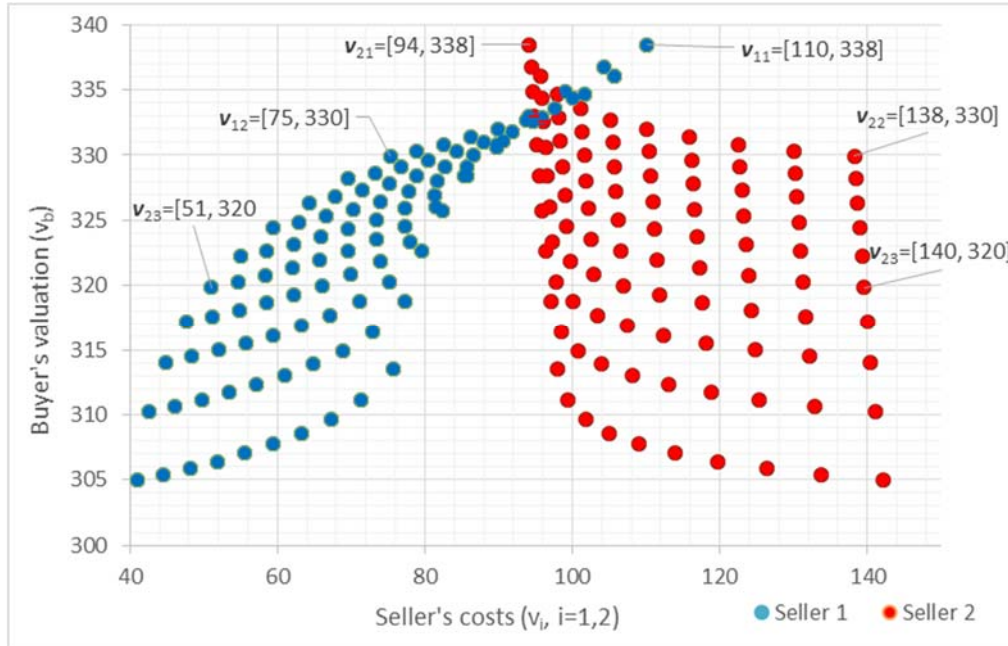
Figure 4. The buyer's valuation and two sellers' costs
(based on Chen-Ritzo, Harrison et al., 2005a)

Seller 1 can increase her surplus and social welfare by offering x_{-1} = [4; 10], which has valuation v_{13} = [48, 317]. In order to win, she needs to reduce the price to 72 so that the buyer's surplus remains 245, but her surplus increases from 10 to 25. Social welfare for v_{13} equals 269 and it is the maximum possible value.

Example 1 shows that, when bounds are imposed, then one seller may have more than one efficient configuration. The efficient configurations are on the boundary of the configuration set (X^{-1}). An increase in attribute levels over the limits changes the efficient configuration and it may change the winning seller. For example, if x_1 level is between 1 and 35 and x_2 level remains between 1 and 10, then Seller 2 wins the auction because $v_1(35, 1) = 370$; $v_2(35, 1) = 91$; and $v_b(11, 1) = 365$; the maximum buyer's surplus is $u_b(35, 1, 91) = 274$. If, however, x_1 levels remain between 1 and 10 and x_2 can take a level between 1 and 20, then Seller 1 wins the auction because $v_1(10, 20) = 41$; $v_2(10, 20) = 266$; and $v_b(11, 1) = 327$; the maximum buyer's surplus is $u_b(10, 20, 41) = 286$.

4.3 Price, configuration and cost

Arguably, the cornerstone of the quasi-linear preference use in multi-attribute auctions is the independence of configuration and price. In an early paper on the subject Che (1993) observed that the set of efficient configurations is the solution of the following problem (obtained from (9) after using (8) to substitute utility with valuation):

$$(i^*, x_{-1}^*) = \operatorname*{arg\,max}_{i \in I, x_{-1} \in X^{-1}} \left(v_b(x_{-1}) - v_i(x_{-1}) \right). \tag{15}$$

In the same proposition (op. cit. p. 673), Che proved that the price corresponding to a dominant strategy equilibrium is equal to costs in second-score auctions and "costs plus" in first-score auctions (e.g., sealed bid), i.e.,

$$x_1^* = v_i(\boldsymbol{x}_{-1}^*) + \rho, \tag{16}$$

where $\rho = 0$ in a second-score auction and $\rho > 0$ in a first-score auction (calculation of ρ is not relevant to this discussion; it can be found in, among others, David, Azoulay-Schwartz et al., 2006).

This proposition is based on the implicit assumption that costs may be used to determine price but not vice versa. This may be the case in markets in which buyers choose goods that had been produced prior to transactions, therefore their costs are fixed at the time of exchange. The role of the allocation mechanism is to determine the optimal allocation of a given set of goods for a given set of buyers. The assumption that price is independent of valuations, including costs, may approximate reality; production and sales are disconnected, thus the sellers' valuation (costs) cannot be related to price.

In procurement, goods may be produced after the contract is awarded. This is especially true in cases involving services which are "configured" during the transaction process. Multi-attribute reverse auctions allow the sellers to determine the buyers' requirements and the price so that they can produce their goods accordingly. They offer different configurations at different prices and they cannot disassociate price from the configuration. Sellers may modify a configuration in order to reduce the price and vice versa. Observe that every example discussed in Section 2 involved goods and services which were not available at the time of the auction.

Configurations differ in their costs; sellers may offer any configuration (technologically feasible) as long as it is profitable. This means that the seller offers a configuration, with a price which is not lower than its costs. Seller's i utility from configuration $\boldsymbol{x}_{-1}$ is given by (2), i.e., $u_i(\boldsymbol{x}) = x_1 - v_i(\boldsymbol{x}_{-1})$. If the sellers are concerned with profit (utility), then they have to base their bidding decisions on (16) rather than (15)—for them configuration and price attributes are interdependent. While they try to keep profit as high as possible, the auction mechanism "pushes" them towards solution (15) because it requires that consecutive bids increase the buyer's valuation $v_b(\boldsymbol{x}_{-1})$ and/or decrease price, which is associated with costs. As an illustration, consider two-attribute A+B auctions used for the procurement of highway construction work discussed in Section 2. Lewis and Bajari (2011) compared A (price-only) and A+B (price and completion time) auctions conducted by California Department of Transportation. They show that the average price in auctions of type A was $1 million lower than in type A+B but the user gain ($d\times$B) was $5.6 million greater in A+B than in A thanks to the fact that work was completed 30-40% faster. The buyer's gain of $4.6 million per contract is statistically significant (op. cit., p. 1194). These results show that two-attribute auctions are better for the buyer and the social welfare than the price-only auctions. They are not worse for the sellers who in A+B auctions charge on average $1 million more than in A so that they can complete work in fewer days. In other words, the sellers bid different prices for different configurations.

The price-costs interdependence may also be present in price-only auctions. Although in such auctions the goods may be considered homogenous by the buyer, the sellers may be able to change materials, technologies and schedules in order to lower production costs. These changes may not affect the specification formulated by the buyer but they may modify attributes which are not specified. In this situation the sellers' utility is not quasi-linear, therefore improvements discussed in Section 5 may be possible.

4.4 Additive scoring functions

Preference elicitation and the construction of a utility function is a process that may be time-consuming and difficult for procurement and sales managers. A simpler and often adequate practice is the construction of a scoring function which approximates utility. The use of scoring

functions has been suggested in the design of multi-attribute auction mechanisms (Branco, 1997; Bichler and Kalagnanam, 2005; Asker and Cantillon, 2008). Scoring functions can be obtained using one of the well-known preference elicitation methods (for an overview see Bichler and Kalagnanam, 2005).

For the purpose of constructing an additive scoring function it is sufficient to assume that: (1) the N-dimensional space of alternatives X is discrete; and (2) the scores assigned to the attributes and to their levels reflect the decision-maker's preferences. Additive scoring function assumes also that attributes are pairwise preferentially interdependent. That is, the value of tradeoffs between any two attributes does not depend on the levels of the remaining attributes (Keeney and Raiffa, 1976).

Preference elicitation methods associate attribute levels, which may be non-numerical, with their preferential equivalents, which are numerical and thus allow tradeoffs between attributes and between attribute levels. The result is an additive scoring function $s(.)$:

$$s(\boldsymbol{x}_l) = \sum_{j \in J} w_j \ s_j(x_{lj}) = \sum_{j \in J} w_j \ s_{lj}, \tag{17}$$

where: $\boldsymbol{x}_l = [x_{l1}, \dots, x_{lN}] \in X$ is alternative l ($l \in L = |X|$), w_j is the weight of attribute j, and $s_j(x_{lj}) = s_{lj}$ is the rating (score) of the level x_{lj} of the attribute j ($j \in N$).

There are three practical problems, which are important for the efficiency of auction mechanisms:

1. The scoring function is not quasi-linear;

2. The non-price component, i.e., the valuation scoring function, is not concave (convex); and

3. The preferential order of the alternatives differs between market participants.

The scoring function is not quasi-linear when preference of money (price variable) is not equal to money. In theory, this may be easily addressed; the participants are allowed to use preference elicitation only on the non-price attributes (Asker and Cantillon, 2008). The price variable is added after the elicitation process is completed. While some participants may find such separation of price awkward, there also may be another issue pertaining to the measurement of the non-price component and assuring that it approximates a concave valuation function. In auctions, quasi-linearity requires that this component be measured in money so that price can be added or subtracted. Elicitation methods do not guarantee that the individual scores, which correspond to attribute levels, are measured in money. Therefore, the participants must use financial measures, which, however, must exclude the price.

Participants may be reluctant to disregard price during preference elicitation; an argument can be made that the participants' consideration of both price and non-price attributes is often associated with the differences in their perspective on price. Buyers who base their purchasing decisions on the long-run price and other direct and indirect costs or on the TCO model, consider many attributes. The middle- or long-term perspective lends itself to associating money with time, which includes future interest paid and various types of risk (e.g., delayed or not delayed payment, litigation, and change in interest). Because different participants are likely to have different financial, market and production constraints their preferences over money may also differ.

4.5 Preferential order

The marginal value of the buyer's and the sellers' valuations is assumed to be positive (i.e., the first derivative is positive). However, there are situations in multi-attribute auctions when this assumption does not hold because of differences in the participants' preferential order. This

means that even if the participants' utilities are quasi-linear over the set of alternatives, the alternatives are differently ordered. Such a situation may be due to the buyer's satiation point. Consider an example of car purchasing, which Parkes and Kalagnanam (2005) use to illustrate a multi-attribute auction mechanism, and assume that there are two attributes: price and speed. An institutional buyer prefers to pay less for cars than more and get faster cars but up to a point. For safety and the total cost of ownership reasons she prefers slower cars over those that exceed a certain maximum speed. This means that the marginal value after the satiation point is reached decreases. Another example is a company which wants a shorter delivery time of a machine rather than longer, but the next-day delivery is worse (more expensive) due to the costs of preparing the site in a short time.

Attributes used in an auction may be nominal, e.g., color, location, and feature set. The preferential order of the levels of such attributes may differ between the buyer and the sellers and among the sellers. Consequently, the participants' utility functions may be quasi-linear over a set of alternatives which have different preferential ordering. Such a situation means that an auction may lead to an efficient solution but be allocative inefficient.

To illustrate this problem a modified example given in Ariba's patent is used (Wellman, 2005, sheets 1 & 4). The data, given in Table 1, describes three attributes and their levels, as well as the rating associated with two non-monetary attributes (Design and Feature). The levels of the attribute Price are not given a score; price levels are assumed to have the same level for everyone. Scores for Design and Feature are expressed in money. Note the difference in the preferential order of attribute Feature for the buyer and for the seller.

Table 1. The buyer's and the seller's attributes, their levels and scores

Attribute	Level	Score (in $)	
		Buyer	**Seller 1**
Price	70	70	70
	80	80	80
	90	90	90
Design	Enhanced	80	35
	Basic	60	21
Feature	a	30	10
	b	48	30
	c	40	10
	d	20	35

The set of feasible alternatives for all possible configurations of the three attributes in the scoring space is shown in Figure 5. The scores are calculated in the way that is typical for multi-attribute auctions, that is, the buyer's score is: $s_b = s_{b, \text{Design}} + s_{b, \text{Feature}} - Price$. For Seller 1, the score is: $s_1 = Price - s_{1, \text{Design}} - s_{1, \text{Feature}}$.

The efficient frontier comprises six alternatives (A to F), however, only B, C and D, maximize social welfare which is $73. There are other efficient alternatives (i.e., A, E and F), for which social welfare is lower than $73. This shows that some efficient alternatives maximize the total score and some do not.
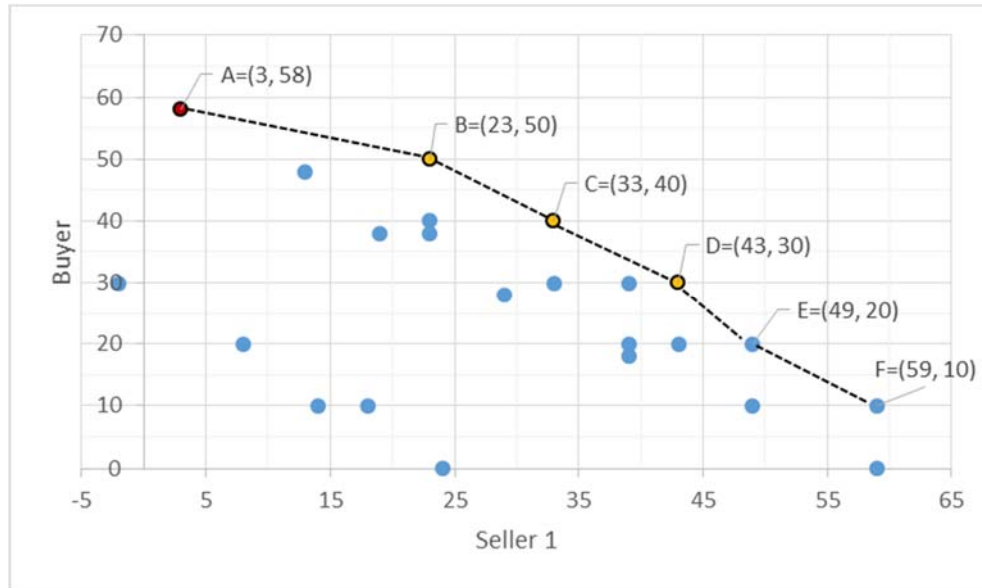
Figure 5. The buyer's and the seller's scores of alternative product configurations

Achieving social welfare equal to $73 through an auction may not be possible. The reasoning is as follows: in an auction, in addition to Seller 1, there are other participating sellers. One of them may submit a bid that yields the buyer's surplus equal to $57. This forces Seller 1 to submit bid A which yields a profit of $3 for this seller and $58 for the buyer. If there is no other bid, then Seller 1 wins; the result is an efficient solution for which social welfare is $61 rather than $73.

The winning bid gives the buyer the highest surplus ($58) from among all the alternatives. However, because the utility is transferable, it might be possible to increase this surplus and at the same time increase the winning seller's surplus. For example, Seller 1 could propose to select alternative B and to share the difference with the buyer; the seller could offer the buyer $12 thus increasing the buyer's surplus from $58 to $70. This would lower the seller's surplus from $23 to $11, which is higher than the winning bid's surplus of $3.

5. Improvements of social welfare and winning bids

Studies of utilities used in trading decisions by firms confirm that often the quasi-linearity assumption is not met (Kim and Reinschmidt, 2010; Campo, 2012). Implications of the assumption and its limitations illustrated with examples (Section 4) show its restrictiveness. This sections discusses the efficient frontiers of the buyer-seller pairs and suggests a mechanism which allows to improve both the winning bids and social welfare.

5.1 Utility

Three typical utility functions discussed in decision analysis literature are concave, linear and convex. Often additive utilities are used because they are simpler to construct and can approximate more general shapes with good accuracy (Keeney and von Winterfeldt, 2007). Mumpower (1991) studied bilateral negotiations and compared the impact of the shape of an additive utility on the settlement space and the efficient frontier. He considered the pairs of utility

functions defined on the bounded set of attributes.

Five feasible sets in the buyer-seller utility space are shown in Figure 6; their efficient frontier is indicated by black line. The first three sets are defined by, respectively, linear/linear pair of utilities, concave/concave pair, and linear/concave pair. These sets have concave or quasi-concave efficient frontier and they are considered here. Note that if the buyer and the sellers have linear utilities, then their preferences over the attributes must differ for the efficient frontier to be quasi-concave. The convex/convex and concave/convex pairs result in the efficiency frontier that is a composition of, respectively, two convex and two concave functions. For comparison, the efficient frontier for quasi-linear utilities is shown by a dotted line.

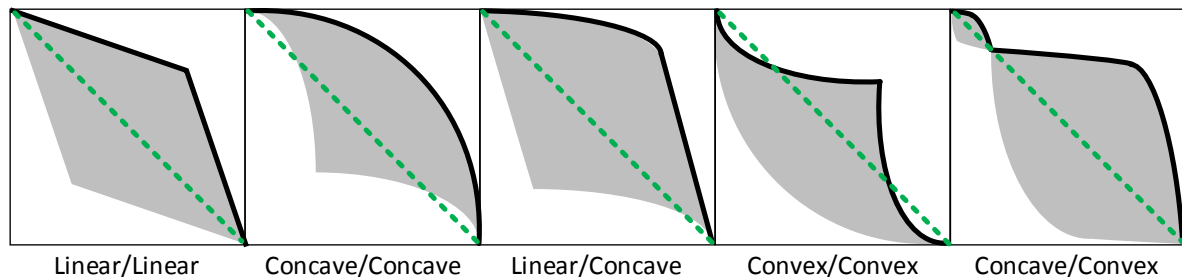| Linear/Linear | Concave/Concave | Linear/Concave | Convex/Convex | Concave/Convex |

Figure 6. Feasible sets and efficient frontiers in utility space (based on Mumpower, 1991)

Mumpower (1991, p. 1309) notes that, because of the different shapes of the efficient frontiers, only the use of multiple strategies leads to settlements which are efficient, maximize social welfare (sum of utilities), and preserve equality. Maximization of the buyer's surplus rather than equality is sought in auctions. However, the buyer's surplus criterion in conjunction with the solution and mechanism efficiency can be met only when the efficient frontier is an interval (see Figure 3). Following Mumpower's insight, we posit that, in situations such as illustrated in Figure 6, the auction protocol needs to be augmented.

5.2 Concave efficient frontier

Raiffa, Richardson and Metcalfe (2003, p. 249-268) present a problem in which Amstore, a retail firm, needs to build a new store and seeks a contractor. Kersten, Vahidov, and Gimon (2013) adapted the Amstore-Nelson case for a procurement context. In the case, Milika, a milk producer, is seeking a logistics service provider for the transportation of milk from a single depot to multiple customers. Several providers participate in the auction set up by Milika, which aims at determining levels of three discrete attributes: (1) standard rate of transportation, (2) rush rate for unexpected delivery, and (3) penalty for the non-delivery or delivery of spoiled goods. Each attribute has fifteen levels, which results in the total of 3375 alternatives.

An example of a set of feasible alternatives in the buyer-seller utility space is shown in Figure 7. The set of alternative contracts is discrete and it can be approximated by a convex set. This is typical for most types of utility functions considered in negotiation analysis (see, e.g., Raiffa, 1982; Young, 1991; Raiffa, Richardson et al., 2003).

When the feasible set is convex, then the efficient frontier is a concave function. The direction for social welfare maximization (joint improvement) is then North-East. This is the direction that the negotiation participants should take if they wish to achieve the, so called, win-win agreement, which also maximizes social welfare. By contrast, in auctions the direction that the bidders take is North-West (when the buyer's utility is on the vertical axis). The result is that the winning bid may

be efficient but it neither maximizes the buyer's surplus nor is it allocative efficient.
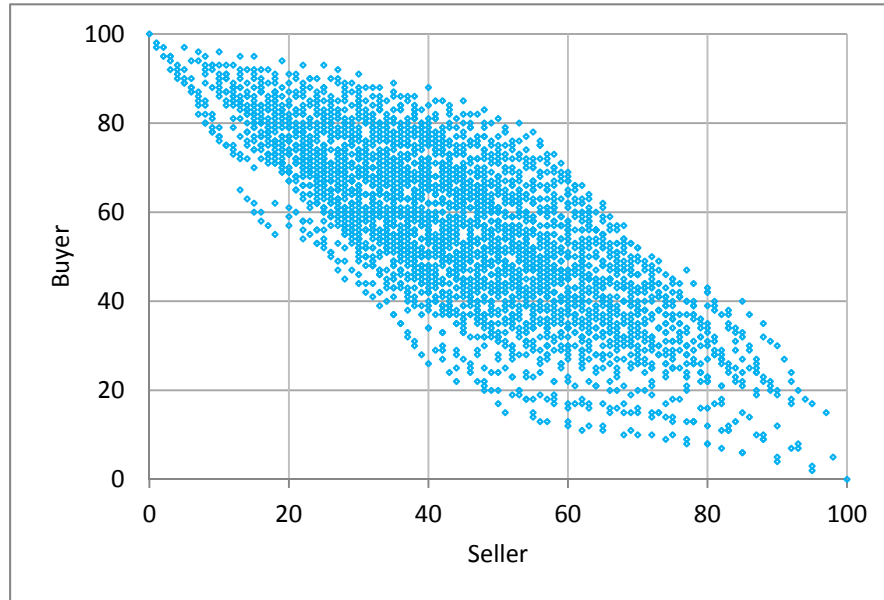

Figure 7. Feasible set in the scoring space of the buyer and one seller.

<u>Proposition 3</u>. If the efficient frontier is continuous and concave, then the efficient winning bid can either maximize the buyer's surplus or be allocative efficient but not both.

<u>Proof</u>. Assume that x_1 is the efficient winning bid made by bidder i and it maximizes the buyer's surplus, i.e., $u_b(x_1) = \max_{x \in X} u_b(x)$, and let $u_1 = (u_b(x_1); u_i(x_1))$. Similarly, assume that x_2 maximizes the winning bidder's surplus, i.e., $u_i(x_1) = \max_{x \in X} u_i(x)$ and let $u_2 = (u_b(x_2); u_i(x_2))$. Because the efficient frontier is continuous it has more than one point, therefore, $u_1 \neq u_2$. The gradient $\nabla(u_b(x) + u_i(x))$ lies in between gradients $\nabla u_b(x)$ and $\nabla u_i(x)$, therefore its direction is towards the point in-between u_1 and u_2. Let $u^* = (u_b(x^*), u_i(x^*))$ be the efficient point in the direction of $\nabla(u_b(x) + u_i(x))$. From the efficient frontier concavity it follows that $u_b(x^*) \neq u_b(x_1)$. By design u^* maximizes social welfare and it is different from u_1 at which buyer's surplus is maximized. ♦

The implication of Proposition 3 is that given a winning bid, which maximizes the buyer's surplus, there is another alternative that maximizes social welfare. However, this may occur even if the efficient frontier is not concave. One example is when the utilities of the buyer and the seller are convex but the social welfare of the alternative that is closest to the utopia point is greater than the one obtained from the winning bid. This case is illustrated in Figure 6. This situation may also occur, as it is shown in Figure 6, when the utilities are concave/convex.

5.3 The value of competition

A suggestion was made in Section 4.5 that in addition to contracts which increase social welfare, albeit at a cost to the buyer, there may also be contracts which increase both social welfare and the buyer's surplus. This is due to the synergy that occurs when the buyer's and the seller's needs are met: a decrease in the buyer's utility may be more than offset by an increase in the seller's utility.

Figure 8 illustrates a common solution (*A*) for both quasi-linear utilities and utilities for which the efficient frontier is concave. Let's assume that seller *i* is the winner and the winning bid is point *A*.

If the preferences are quasi-linear, then *A* maximizes both the buyer's surplus and social welfare. If the efficient frontier is concave, then *A* does not maximize social welfare; both *B* and *D* yield a higher social welfare than *A*.
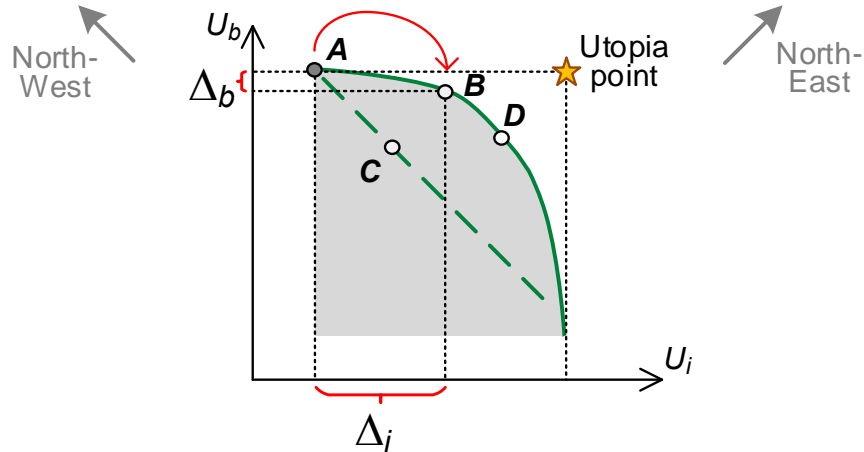

Figure 8. Improvement of the winning bid A to *B* for concave efficient frontier

Market participants who want to maximize social welfare need to move in the North-East direction. Sellers, who are pushed by competition to increase the buyer's surplus, move in the North-West direction. Quasi-linear preferences together with the use of the sum of utilities as the measure of social welfare, remove the conflict in directions because the North-West moves do not change the distance from the Utopia point (max u_b; max u_i). However, market participants should be aware of the conflict as it arises when other types of preferences and/or other welfare measures are deemed more suitable.

The alternatives shown in Figure 8 have the following coordinates (u_b, u_i): *A* = (17; 3); *B* = (16; 11); *C* = (12; 7.5); and *D* = (13; 14.5). If we move from *A* to *B*, then social welfare increases from 20 to 27, i.e., by 35%. The maximum social welfare is 27.5 and it is reached at alternative *D*. This simple example illustrates that the difference in social welfare value may be significant.

Moreover, reaching a solution which is better than the winning bid may be possible. This, however, requires moving beyond the initial problem formulation. Let's assume that u_b and u_i are both expressed in monetary terms. We can see that the move from *A* to *B* results in buyer *b*'s loss $(u_b(A) - u_b(B)) = \nabla_b = \$1$ and seller *i*'s gain $(u_i(B) - u_i(A)) = \nabla_i = \$8$. If buyer *b* realizes the differences between winning bid *A* and alternative *B*, then she could suggest selecting alternative *B* under the condition that *i* pays her \$5 (or some other amount, which exceeds \$1).

When the utilities are not quasi-linear, then price transfer affects welfare. These utilities may also be assumed to be transferable (this assumption is often made in economics). If the amount to be transferred is positive, i.e., $\nabla_i - \nabla_b > 0$, then the winning bid *A* may be improved.

When the efficient frontier is concave, the move from the winning bid (*A* in Fig. 8) to a bid that increases social welfare (*B*) involves a transfer which is similar to price transfer when the frontier is linear with -1 slope, except for the following three differences:

1. Both price and configuration are included in the transfer and in social welfare calculation;

2. The transfer of value from the seller to the buyer requires change of the configuration; and

3. The value transfer improves the buyer's and the seller's surplus as well as the social welfare.

Value θ that is transferred to the buyer has to be greater than the buyer's loss (i.e., $\theta > \nabla_b$). The difference between seller's i gain and buyer's b loss, i.e., $\nabla_i - \nabla_b > 0$, corresponds to the social welfare increase. Assuming that the concavity of the efficient frontiers for the pairs buyer$_b$-seller$_i$ ($i \in I$) is given and does not change, the size of the transferable value ($\nabla_i - \nabla_b$) can be viewed as the "value of competition". This is because the stronger the competition the greater the buyer's surplus, that is, the winning bid is further from the solution maximizing social welfare. To ascertain this let's denote the concave efficient frontier as function of u_i, i.e., $\upsilon_b(u_i)$ and assume that $\upsilon_b(u_i)$ is twice differentiable. We also assume that the buyer's utility produced by the winning bid is not smaller than the utility which maximizes social welfare.

Proposition 4. Given concave efficient frontier, the greater the utility value of the buyer for the winning bid the greater the transferable value ($\nabla_i - \nabla_b$) is.

Proof: Function $\upsilon_b(u_i)$ is concave, therefore its second derivative is non-positive ($\upsilon''_b \leq 0$). This means that the speed of increase of $\upsilon_b(u_i)$ is decreasing with the increase of u_i. Conversely, the speed of the increase of $\upsilon_b(u_i)$ increases when the value u_i decreases. In other words, a small change in u_i causes an increasingly greater change in $\upsilon_b(u_i)$ as u_i gets smaller. ♦

6. Discussion and conclusions

If the buyers' and the sellers' preferences are quasi-linear, then multi-attribute reverse auctions are efficient mechanisms producing efficient solutions which maximize the buyers' utility. If they are not, then the sellers compete for the contract steering the process towards a solution which maximizes the buyer's utility but not social welfare. However, we have shown in Section 4.5 that even when the preferences are quasi-linear, auctions may be inefficient.

Goods and services sourced by organizations are often multi-attribute, with the sellers being capable of providing different configurations. A review of several multi-attribute reverse auctions given in Section 2, gives grounds for concern about this type of auctions considered to be an efficient mechanism. An in-depth analysis of the quasi-linearity assumption and its implications given in Section 3 and its limitations given in Section 4 confirms Ausubel and Milgrom's (2006) observation that this assumption is frequently violated.

Concave and linear utilities result in a concave efficient frontier. In this case, the auctions which have been discussed in literature, are likely inefficient. They result in an efficient solution that maximizes the buyer's surplus which, however, is different from the efficient solution that maximizes social welfare. This difference may mean a significant social welfare loss. In a simple illustrative example presented in Section 4.5 the loss is over 35%. For many firms, especially public organizations, this may be an important indication of auctions' limitation.

The fact that auctions do not maximize social welfare could be of no interest for private business, if not for the unrealized gains. When the efficient frontier is concave, then the winning bid obtained through an auction can be improved for both the buyer and the winning seller. Even if the utility is convex on one side (e.g., the buyer) and concave on the other (e.g., the seller), the possibility for joint improvement exists (see Figure 6). This possibility does not depend on whether or not the procurement auction is multi- or single-attribute. In the case of price-only auctions the configuration does not change but a joint improvement is possible when the side exchange (premium) compensates for the differences in the valuation of money.

Section 5 suggests directions for the design of exchange mechanisms, which would be more

flexible than reverse auctions. The mechanisms need to allow the buyers to decide about the preferred direction which will maximize their surplus (i.e., North-West), maximize social welfare (i.e., North-East), or be in-between these two extremes. An example of such a mechanism is a three-phase process. In the first step a reverse auction is conducted. If the auction concludes with a winning bid, then, in the second step, the alternatives which were the bids preceding the winning bid, are selected. For every selected alternative the minimum premium is determined. The premium has to compensate for the loss the buyer incurred because of the replacement of the winning bid with another alternative. In the third step the second reverse auction is conducted; in this auction the selected alternatives are posted and the sellers bid on the premium value.

If the auction is price-only or the utility is transferable, then the three-phase process does not require that the buyer participate in the second step. If the utility is non-transferable, then the buyer has to determine the required minimum premium for each alternative presented in the third step.

This process can also be used irrespectively of the participants' preferences. If the efficient frontier is convex or linear, then no bidder should be willing to enter the second auction. This is because the sellers should not propose a premium that would increase the value of an alternative because they would incur additional losses. A seller would propose an alternative which has a lower utility than the utility of the winning bid and then he would have to further decrease this utility by the premium. Only if the efficient frontier is concave the seller may offer a premium for an alternative so that it benefits both the buyer and the seller.

The three-phase process does not assure that the final bid maximizes social surplus. In order to achieve this, utilities of all participants would need to be known. It does, however, allow for the participants to search for alternatives that increase social welfare benefiting both the buyer and the seller. An important requirement of this process is strategy proof. This requirement, as well as the concrete formal representations of the mechanism outlined here, need to be determined and also experimentally studied.

This study presents weaknesses of reverse auctions. It relies on the observations of procurement auctions and on auction theory. Two research directions need to be followed in order to verify the study's practical usefulness. One direction would be the assessment of social welfare losses in real-life auctions. A complementary research direction would be the design and use of augmented mechanisms and the assessment of their contribution to reaching jointly improved solutions that also increase social welfare.

Lastly, risk neutrality and quasi-linearity are also assumed in forward auctions. An analysis of auction models over goods bought in order to achieve profit (e.g., production goods and services) may lead to similar result as these presented here.

References

Adomavicius, G., A. Gupta and P. Sanyal (2012). "Effect of Information Feedback on the Outcomes and Dynamics of Multisourcing Multiattribute Procurement Auctions." *J. of Management Information Systems*, **28**(4): 199-230.

Asker, J. and E. Cantillon (2008). "Properties of Scoring Auctions." *The RAND J. of Economics*, **39**(1): 69-85.

Athey, S. and J. Levin (2001). "Information and Competition in Us Forest Service Timber Auctions." *J. of Political Economy*, **109**(2): 375-417.

Ausubel, L. M. and P. Milgrom (2006). The Lovely but Lonely Vickrey Auction. *Combinatorial Auctions*. P.

Cramton, Y. Shoham and R. Steinberg. Boston, MIT Press**:** 17-40.

Bajari, P. and A. Hortacsu (2005). "Are Structural Estimates of Auction Models Reasonable? Evidence from Experimental Data." *J. of Political Economy*, **113**(4): 703-741.

Beil, D. R. and L. M. Wein (2003). "An Inverse-Optimization-Based Auction Mechanism to Support a Multiattribute Rfq Process." *Management Science*, **49**(11): 1529-1545.

Bellosta, M. J., S. Kornman and D. Vanderpooten (2008). "A Unified Framework for Multiple Criteria Auction Mechanisms." *Web Intelligence and Agent Systems*, **6**(4): 401-419.

Bergstrom, T. C. and H. R. Varian (1985). "When Do Market Games Have Transferable Utility?" *J. of Economic Theory*, **35**(2): 222-233.

Bichler, M., A. Davenport, G. Hohner, et al. (2006). Industrial Procurement Auctions. Combinatorial Auction. P. Cramton, Y. Shoham and R. Steinberg. Boston, MIT Press.

Bichler, M. and J. Kalagnanam (2005). "Configurable Offers and Winner Determination in Multi-Attribute Auctions." *European J. of Operational Research*, **160**(2): 380-394.

Bichler, M. and J. R. Kalagnanam (2006). "Software Frameworks for Advanced Procurement Auction Markets." *Communications of the ACM*, **49**(12): 104-108.

Boyne, G. A. (2002). "Public and Private Management: What's the Difference?" *J. of Management Studies*, **39**(1): 97-122.

Branco, F. (1997). "The Design of Multidimensional Auctions." *The RAND J. of Economics*, **28**(1): 63-81.

Campo, S. (2012). "Risk Aversion and Asymmetry in Procurement Auctions: Identification, Estimation and Application to Construction Procurements." *J. of Econometrics*, **168**(1): 96-107.

Che, Y. K. (1993). "Design Competition through Multidimensional Auctions." *The RAND J. of Economics*, **24**(4): 668-680.

Chen-Ritzo, C. H., T. P. Harrison, A. M. Kwasnica, et al. (2005a). "Better, Faster, Cheaper: An Experimental Analysis of a Multiattribute Reverse Auction Mechanism with Restricted Information Feedback." *Management Science*, **51**(12): 1753-1762.

Chen-Ritzo, C. H., T. P. Harrison, A. M. Kwasnica, et al. (2005b) "Better, Faster, Cheaper: Online Supplement." *Management Science*.

David, E., R. Azoulay-Schwartz and S. Kraus (2006). "Bidding in Sealed-Bid and English Multi-Attribute Auctions." *Decision Support Systems*, **42**(2): 527-556.

El-Rayes, K. and A. Kandil (2005). "Time-Cost-Quality Trade-Off Analysis for Highway Construction." *J. of Construction Engineering and Management*, **131**(4): 477-486.

Ferrin, B. G. and R. E. Plank (2002). "Total Cost of Ownership Models: An Exploratory Study." *J. of Supply Chain Management*, **38**(3): 18-29.

Gupta, A., S. T. Parente and P. Sanyal (2012). "Competitive Bidding for Health Insurance Contracts: Lessons from the Online Hmo Auctions." *International J. of Health Care Finance and Economics*, **12**(4): 303-322.

Gupta, D., E. Snir and Y. Chen (2013). Contractors' and Agency Decisions and Policy Implications in A+B Bidding. Working paper, University of Minnesota.

Gwebu, K. L. (2009). "Decision Support in Multi-Attribute Reverse Auctions " *J. of Electronic Commerce Research*, **10**(4): 252-264.

Hatfield, J. W. and P. Milgrom (2005). "Matching with Contracts." *American Economic Review,* **95**(4): 913-935.

Hohner, G., J. Rich, E. Ng, E. Ng, G. Reid, A. J. Davenport, J. R. Kalagnanam, H. S. Lee and A. Chae (2003). "Combinatorial and Quantity-Discount Procurement Auctions Benefit Mars, Incorporated and Its Suppliers." *Interfaces*, **33**(1): 23-35.

Hurwicz, L. and M. Walker (1990). "On the Generic Nonoptimality of Dominant-Strategy Allocation Mechanisms: A General Theorem That Includes Pure Exchange Economies." *Econometrica*: **58**(3): 683-704.

Kalagnanam, J. and D. C. Parkes (2004). Auctions, Bidding and Exchange Design. *Handbook of Quantitative*

Supply Chain Analysis: Modeling in the E-Business Era. D. Simchi-Levi, S. Wu and Z. Shen. Boston, Kluwer**:** 143-212.

Kameshwaran, S., Y. Narahari, C. H. Rosa, et al. (2007). "Multiattribute Electronic Procurement Using Goal Programming." *European J. of Operational Research*, **179**(2): 518-536.

Karakaya, G. and M. Köksalan (2011). "An Interactive Approach for Multi-Attribute Auctions." *Decision Support Systems*, **51**(2): 299-306.

Keeney, R. L. and H. Raiffa (1976). *Decisions with Multiple Objectives: Preferences and Value Trade-Offs*. New York, Wiley.

Keeney, R. L. and D. von Winterfeldt (2007). Practical Value Models *Advances in Decision Analysis*. E. Wards, R. F. Miles and D. von Winterfeldt. Cambridge Cambridge Univ. Press**:** 232-251.

Kersten, G. E. and S. J. Noronha (1998). "Rational Agents, Contract Curves, and Inefficient Compromises." *IEEE Systems, Man, and Cybernetics*, **28**(3): 326-338.

Kersten, G. E., R. Vahidov and D. Gimon (2013). "Concession-Making in Multi-Attribute Auctions and Multi-Bilateral Negotiations: Theory and Experiments." *Electronic Commerce Research and Applications*, **12**: 166-180.

Kim, H.-J. and K. F. Reinschmidt (2010). "Effects of Contractors' Risk Attitude on Competition in Construction." *J. of construction Engineering and Management*, **137**(4): 275-283.

Krishna, V. (2009). *Auction Theory*, Academic Press.

Lewis, G. and P. Bajari (2011). "Procurement Contracting with Time Incentives: Theory and Evidence." *The Quarterly J. of Economics*, **126**(3): 1173-1211.

Luce, R. D. and H. Raiffa (1957). *Games and Decisions: Introduction and Critical Survey*, Courier Dover Publications.

Mumpower, J. L. (1991). "The Judgement Policies of Negotiators and the Structure of Negotiation Problems." *Management Science*, **37**(10): 1304 - 1324.

Parkes, D. C. and J. Kalagnanam (2005). "Models for Iterative Multiattribute Procurement Auctions." *Management Science*, **51**(3): 435–451.

Parsons, S., J. A. Rodriguez-Aguilar and M. Klein (2011). "Auctions and Bidding: A Guide for Computer Scientists." *ACM Computing Surveys* **43**(2): 10:11-10:59.

Pham, L., A. Zaitsev, R. Steiner and J. E. Teich (2013). "Negotiauction: An Experimental Study." *Decision Support Systems*.

Raiffa, H. (1982). *The Art and Science of Negotiation*. Cambridge, MA, Harvard University Press.

Raiffa, H., J. Richardson and D. Metcalfe (2003). *Negotiation Analysis. The Science and Art of Collaborative Decision Making*. Cambridge, Harvard University Press.

Rothkopf, M. H. (2007). "Decision Analysis: The Right Tool for Auctions." *Decision Analysis*, **4**(3): 167-172.

Sandholm, T. W. (2007). "Expressive Commerce and Its Application to Sourcing: How We Conducted $35 Billion of Generalized Combinatorial Auctions." *AI Magazine*, **28**(3): 45.

Simon, J. and F. Melese (2011). "A Multiattribute Sealed-Bid Procurement Auction with Multiple Budgets for Government Vendor Selection." *Decision Analysis*, **8**(3): 170-179.

Strecker, S. (2010). "Information Revelation in Multiattribute English Auctions: A Laboratory Study." *Decision Support Systems*, **49**(3): 272-280.

Teich, J. E., H. Wallenius, J. Wallenius and A. Zaitsev (2006). "A Multi-Attribute E-Auction Mechanism for Procurement: Theoretical Foundations." *European J. of Operational Research*, **175**(1): 90-100.

Varian, H. R. (2010). *Intermediate Economics. A Modern Approach*. New York, Norton.

Wellman, M. P. (2005). System and Method for Matching Multi-Attribute Auction Bids. U.S., Ariba, Inc.

Young, H. P., Ed. (1991). *Negotiation Analysis*. Ann Arbor, The University of Michigan Press.